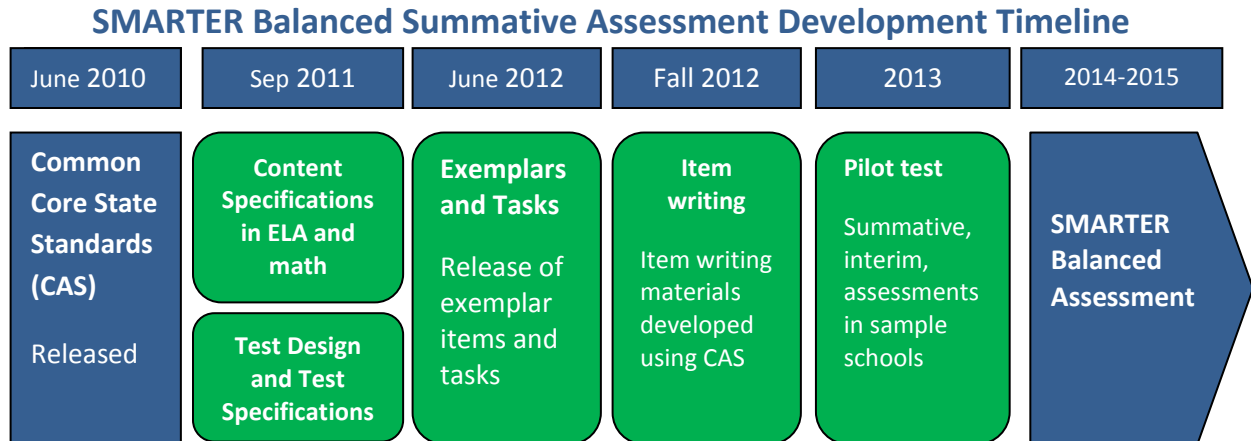


# SMARTER Balanced Assessment High School Mathematics

As 2015 approaches and we transition from Missouri's traditional MAP testing to the Smarter Balanced Assessments, many teachers and administrators have been asking for information. What is the implementation timeline? What kinds of items will be on the test? What do assessment items look like? How are the tests scored? How will technology be utilized? While not every detail is known, information is available.

First, an implementation time line:



From <http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/ItemSpecifications/GeneralItemSpecifications.pdf>

## Item Types

SBAC assessments are made up of four item types: Selected-Response, Constructed-Response, Technology-Enhanced, and Performance Task. A description of those items follows.

### Selected-Response Items (SR)

Traditionally known as multiple choice, selected-response items include a stimulus and stem followed by three to five options from which a student is directed to choose only one.

### Constructed-Response Items (CR)

The main purpose of a constructed-response item is to address targets and claims that are of greater complexity. They ask students to develop answers without suggested answer choices.

### Technology-enhanced Items/Tasks (TE)

Technology-enhanced items can provide evidence for mathematics practices that could not be as reliably obtained from traditional SRs and CRs. Technology-enhanced items may stand alone or may be a tool used as part of the Performance Task and/or Constructed-Response items.

### Performance Tasks (PT)

Performance tasks, the most complex of all items, include the following elements:

- Integrate knowledge and skills across multiple claims.
- Measure capacities such as depth of understanding, research skills, and/or complex analysis with relevant evidence.
- Require student-initiated planning, management of information/data and ideas, and/or interaction with other materials.
- Reflect a real-world task and/or scenario-based problem.

- Allow for multiple approaches.
- Represent content that is relevant and meaningful to students.
- Allow for demonstration of important knowledge and skills.
- Require scoring that focuses on the essence of the Claim(s) for which the task was written.
- Seem feasible for the school/classroom environment.

## Claims

The Smarter Balanced summative assessments in mathematics are designed to measure the full range of student abilities in the Common Core State Standards or Core Academic Standards (CAS). Evidence will be gathered in support of four major claims: (1) Concepts and Procedures, (2) Problem Solving, (3) Communicating Reasoning, and (4) Modeling and Data Analysis. Students will receive an overall mathematics composite score. For the enhanced assessment, students will receive a score for each of three major claim areas. (Math claims 2 and 4 are combined for the purposes of score reporting.)

**Claim 1** — Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

**Claim 2** — Students can solve a range of complex, well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

**Claim 3** — Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

**Claim 4** — Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

## Glossary

**Distracter:** the incorrect response options to an SR item.

**Distracter Analysis:** the item writer’s analysis of the options or rationale for inclusion of specific options.

**Item:** the entire item, including the stimulus, question/prompt, answer/options, scoring criteria, and metadata.

**Key:** the correct response(s) to an item.

**Options:** the responses to a selected-response (SR) item from which the student selects one or more answers.

**Scoring Rubric:** the descriptions for each score point for an item/task that scores more than one point for a correct response.

**Stem:** the statement of the question or prompt to which the student responds.

**Stimulus:** the text, source (e.g., video clip), and/or graphic about which the item is written. The stimulus provides the context of the item/task to which the student must respond.

**Task:** similar to an item, yet typically more involved and usually associated with constructed-response, extended-response, and performance tasks.

**Top-Score Response:** one example of a complete and correct response to an item/task.

**Additional information** (including Scoring Rubrics) is available at:

<http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/03/DRAFTMathItemSpecsShowcase2.pdf>

<http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/ItemSpecifications/GeneralItemSpecifications.pdf>

**Additional Sample Items are available:**

[3<sup>RD</sup> GRADE SAMPLE  
ELA ITEMS](#)

[4<sup>TH</sup> GRADE SAMPLE  
ELA ITEMS](#)

[5<sup>TH</sup> GRADE SAMPLE  
ELA ITEMS](#)

[6<sup>TH</sup> GRADE SAMPLE  
ELA ITEMS](#)

[7<sup>TH</sup> GRADE SAMPLE  
ELA ITEMS](#)

[8<sup>TH</sup> GRADE SAMPLE  
ELA ITEMS](#)

[9<sup>TH</sup> GRADE SAMPLE  
ELA ITEMS](#)

[10<sup>TH</sup> GRADE SAMPLE  
ELA ITEMS](#)

[11<sup>TH</sup> GRADE SAMPLE  
ELA ITEMS](#)

[3<sup>RD</sup> GRADE SAMPLE  
MATHEMATICS ITEMS](#)

[4<sup>TH</sup> GRADE SAMPLE  
MATHEMATICS ITEMS](#)

[5<sup>TH</sup> GRADE SAMPLE  
MATHEMATICS ITEMS](#)

[6<sup>TH</sup> GRADE SAMPLE  
MATHEMATICS ITEMS](#)

[7<sup>TH</sup> GRADE SAMPLE  
MATHEMATICS ITEMS](#)

[8<sup>TH</sup> GRADE SAMPLE  
MATHEMATICS ITEMS](#)

## MAT.HS.CR.1.00FBF.N.275

|  |   |
|--|---|
| Sample Item ID:  | MAT.HS.CR.1.00FBF.N.275   |
| Grade:   | HS  |
| Claim(s):  | <b>Claim 1: Concepts and Procedures</b><br>Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency. |
| Assessment Target(s):                                    | 1 N: Build a function that models a relationship between two quantities.  |
| Content Domain:  | Functions   |
| Standard(s):   | F-BF.2  |
| Mathematical Practice(s):                                | 1, 2, 4, 7  |
| DOK:   | 2   |
| Item Type:   | CR  |
| Score Points:  | 1   |
| Difficulty:  | M   |
| Key:   | $f(n) = \frac{3}{2} \cdot f(n-1)$ for $n > 1$ , where $f(1) = 8$  |
| Stimulus/Source:   |   |
| Target-specific attributes (e.g., accessibility issues): |   |
| Notes:   | Equation editor (or some equivalent functionality) will need to be available.   |

The first four terms of a sequence are shown below.

8, 12, 18, 27, ...

Write a recursive function for this sequence.

*Sample Top-Score Response:*

Correct responses to this item will receive 1 point.

1 point:  $f(n) = \frac{3}{2} \cdot f(n-1)$  for  $n > 1$ , where  $f(1) = 8$

## MAT.HS.CR.1.00FBF.N.276

|  |   |
|--|---|
| Sample Item ID:  | MAT.HS.CR.1.00FBF.N.276   |
| Grade:   | HS  |
| Claim(s):  | <b>Claim 1: Concepts and Procedures</b><br>Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency. |
| Assessment Target(s):                                    | 1 N: Build a function that models a relationship between two quantities.  |
| Content Domain:  | Functions   |
| Standard(s):   | F-BF.2  |
| Mathematical Practice(s):                                | 1, 2, 4, 7  |
| DOK:   | 2   |
| Item Type:   | CR  |
| Score Points:  | 1   |
| Difficulty:  | M   |
| Key:   | $f(n) = 24,500 - 4900n$   |
| Stimulus/Source:   |   |
| Target-specific attributes (e.g., accessibility issues): |   |
| Notes:   | Equation editor (or some equivalent functionality) will need to be available.   |

A company purchases \$24,500 of new computer equipment. For tax purposes, the company estimates that the equipment decreases in value by the same amount each year. After 3 years, the estimated value is \$9800.

Write an explicit function that gives the estimated value of the computer equipment  $n$  years after purchase.

*Sample Top-Score Response:*

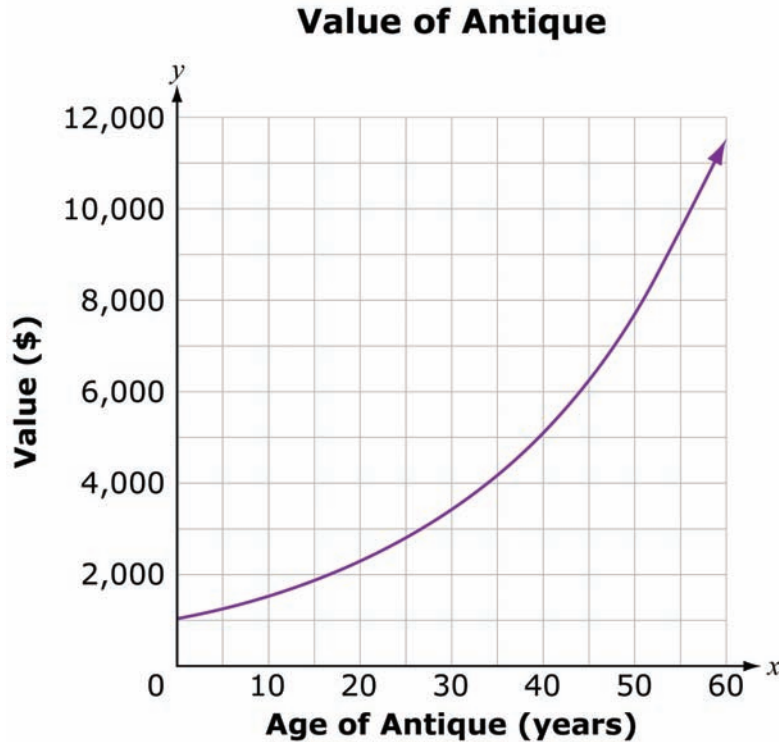
Correct responses to this item will receive 1 point.

1 point:  $f(n) = 24,500 - 4900n$

MAT.HS.CR.1.00FIF.L.614

|  |  |
|--|--|
| Sample Item ID:  | MAT.HS.CR.1.00FIF.L.614  |
| Grade:   | HS   |
| Claim(s):  | <b>Claim 1: Concepts and Procedures</b><br>Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.  |
| Assessment Target(s):                                    | 1 L: Interpret functions that arise in applications in terms of a context.   |
| Content Domain:  | Functions  |
| Standard(s):   | F-IF.6   |
| Mathematical Practice(s):                                | 2, 4, 6  |
| DOK:   | 1  |
| Item Type:   | CR   |
| Score Points:  | 2  |
| Difficulty:  | L  |
| Key:   | 100; 150   |
| Stimulus/Source:   |  |
| Target-specific attributes (e.g., accessibility issues): |  |
| Notes:   | The values in the graph were specifically chosen so that if a student understands how to find average rate of change, no matter how they (reasonably) estimate values from years 0 and 20 and 20 and 40, his/her rounding should come out to the correct answer. |

The value of an antique has increased exponentially, as shown in this graph.



Based on the graph, estimate to the nearest \$50 the average rate of change in value of the antique for the following time intervals:

from 0 to 20 years      \$

from 20 to 40 years      \$

*Scoring Rubric:*  
 Each item is scored independently and will receive 1 point.  
 1 point for the correct estimated average rate from years 0 to 20: \$100

HS Mathematics Sample CR Item C1 TL

1 point for the correct estimated average rate from years 20 to 40: \$150



## MAT.HS.CR.1.00FIF.M.274

|  |   |
|--|---|
| Sample Item ID:  | MAT.HS.CR.1.00FIF.M.274   |
| Grade:   | HS  |
| Claim(s):  | <b>Claim 1: Concepts and Procedures</b><br>Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency. |
| Assessment Target(s):                                    | 1 M: Analyze functions using different representations.   |
| Content Domain:  | Functions   |
| Standard(s):   | F-IF.8  |
| Mathematical Practice(s):                                | 1, 7  |
| DOK:   | 1   |
| Item Type:   | CR  |
| Score Points:  | 1   |
| Difficulty:  | L   |
| Key:   | $y = \frac{2}{3}x + \frac{1}{3}$  |
| Stimulus/Source:   |   |
| Target-specific attributes (e.g., accessibility issues): |   |
| Notes:   |   |

Write the function  $y - 3 = \frac{2}{3}(x - 4)$  in the equivalent form **most** appropriate for identifying the slope and  $y$ -intercept of the function.

*Scoring Rubric for Multi-part Items:*

Correct responses to this item receive 1 point.

1 point for the correct form  $y = \frac{2}{3}x + \frac{1}{3}$

HS Mathematics Sample CR Item Claim 2

**MAT.HS.CR.2.OACED.A.225**

|  |  |
|--|--|
| Sample Item ID:  | MAT.HS.CR.2.OACED.A.225  |
| Grade:   | HS   |
| Primary Claim:   | <b>Claim 2: Problem Solving</b><br>Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.   |
| Secondary Claim(s):                                      | Claim 1: Concepts and Procedures<br>Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.   |
| Primary Content Domain:                                  | Algebra  |
| Secondary Content Domain(s):                             |  |
| Assessment Target(s):                                    | 2 A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.<br><br>2 D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).<br><br>1 G: Create equations that describe numbers or relationships.<br><br>1 D (Gr 8): Analyze and solve linear equations and pairs of simultaneous linear equations.<br><br>1 D (Gr 7): Solve real-life and mathematical problems using numerical and algebraic expressions and equations. |
| Standard(s):   | A-CED.2, 8.EE.7, 7.EE.4  |
| Mathematical Practice(s):                                | 1, 4, 7  |
| DOK:   | 2  |
| Item Type:   | CR   |
| Score Points:  | 3  |
| Difficulty:  | M  |
| Key:   | See Sample Top-Score Response.   |
| Stimulus/Source:   | For photo book prices and sizes:<br><a href="http://www.snapfish.com/snapfish/fe/photo-books">http://www.snapfish.com/snapfish/fe/photo-books</a><br><a href="http://www.shutterfly.com/photo-books/custom-path">http://www.shutterfly.com/photo-books/custom-path</a>   |
| Target-specific attributes (e.g., accessibility issues): |  |
| Notes:   | Requires AI scoring.   |

David compares the sizes and costs of photo books offered at an online store. The table below shows the cost for each size photo book.

| Book Size        | Base Price | Cost for Each Additional Page |
|------------------|------------|-------------------------------|
| 7-in. by 9-in.   | \$20       | \$1.00                        |
| 8-in. by 11-in.  | \$25       | \$1.00                        |
| 12-in. by 12-in. | \$45       | \$1.50                        |

The base price reflects the cost for the first 20 pages of the photo book.

1. Write an equation to represent the relationship between the cost,  $y$ , in dollars, and the number of pages,  $x$ , for each book size. Be sure to place each equation next to the appropriate book size. Assume that  $x$  is at least 20 pages.

7-in. by 9-in.

8-in. by 11-in.

12-in. by 12-in.

2. What is the cost of a 12-in. by 12-in. book with 28 pages?

3. How many pages are in an 8-in. by 11-in. book that costs \$49?

*Sample Top-Score Response:*

- |                   |                  |
|-------------------|------------------|
| 1. 7-in. by 9-in. | $y = x$          |
| 8-in. by 11-in.   | $y = x + 5$      |
| 12-in. by 12-in.  | $y = 1.50x + 15$ |
2. \$57
3. 44 pages

*Scoring Rubric:*

*Responses to this item will receive 0-3 points, based on the following:*

**3 points:** The student has a solid understanding of how to make productive use of knowledge and problem-solving strategies to solve a problem arising in everyday life. The student writes equations to model a real-life situation and uses the equations to find answers to questions within a context. The student correctly writes all three cost equations in question 1, and uses the appropriate equations from question 1, or equivalent equations, to solve for the unknown cost in question 2 and the number of book pages in question 3.

**2 points:** The student demonstrates some understanding of how to make productive use of knowledge and problem-solving strategies to solve a problem arising in everyday life. The student writes equations to model the real-life situation in question 1, but does not write correct equations for all three cases. The student, however, demonstrates understanding of how to use the equations to find answers to questions within context. The answers for questions 2 and 3 represent correct calculations that may or may not use incorrect equation(s), or equivalent equations, written for question 1.

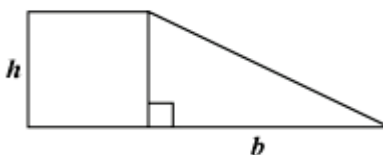
**1 point:** The student has basic understanding of how to make productive use of knowledge and problem-solving strategies to solve a problem arising in everyday life. The student writes equations to model a real-life situation for question 1, with one or more equations containing errors. The student demonstrates partial understanding of how to use the equations to find answers to questions within context. The answers for either question 2 or 3 represent an incorrect calculation using the equations, or equivalent equations, written for question 1.

**0 points:** The student demonstrates inconsistent understanding of how to make productive use of knowledge and problem-solving strategies to solve a problem arising in everyday life. The student is unable to write any correct equation for question 1. The answers to both questions 2 and 3 are incorrect calculations using the equations, or equivalent equations, written for question 1.

MAT.HS.CR.2.0ASSE.A.005

|  |   |
|--|---|
| Sample Item ID:  | MAT.HS.CR.2.0ASSE.A.005   |
| Grade:   | HS  |
| Primary Claim:   | <b>Claim 2: Problem Solving</b><br>Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.  |
| Secondary Claim(s):                                      | Claim 1: Concepts and Procedures<br>Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.  |
| Primary Content Domain:                                  | Algebra   |
| Secondary Content Domain(s):                             |   |
| Assessment Target(s):                                    | 2 A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.<br><br>1 B: Write expressions in equivalent forms to solve problems.<br><br>1 G: Create equations that describe numbers or relationships. |
| Standard(s):   | A-SSE.2, A-CED.4  |
| Mathematical Practice(s):                                | 1, 7  |
| DOK:   | 2   |
| Item Type:   | CR  |
| Score Points:  | 2   |
| Difficulty:  | H   |
| Key:   | $h = \sqrt{80 + \frac{1}{16}b^2} - \frac{1}{4}b$  |
| Stimulus/Source:   |   |
| Target-specific attributes (e.g., accessibility issues): |   |
| Notes:   |   |

The figure below is made up of a square with height,  $h$  units, and a right triangle with height,  $h$  units, and base length,  $b$  units.



The area of this figure is 80 square units.

Write an equation that solves for the height,  $h$ , in terms of  $b$ .  
Show all work necessary to justify your answer.

$h =$  \_\_\_\_\_

*Sample Top-Score Response:*

$$h^2 + \frac{1}{2}bh = 80$$

$$h^2 + \frac{1}{2}bh + \frac{1}{16}b^2 = 80 + \frac{1}{16}b^2$$

$$\left(h + \frac{1}{4}b\right)^2 = 80 + \frac{1}{16}b^2$$

$$h + \frac{1}{4}b = \sqrt{80 + \frac{1}{16}b^2}$$

$$h = \sqrt{80 + \frac{1}{16}b^2} - \frac{1}{4}b$$

*Scoring Rubric:*

*Responses to this item will receive 0-2 points, based on the following:*

**2 points:** The student has a solid understanding of how to solve problems by using the structure of an expression to find ways to rewrite it. The student makes productive use of knowledge and problem-solving strategies by correctly rearranging a formula to highlight a quantity of interest.

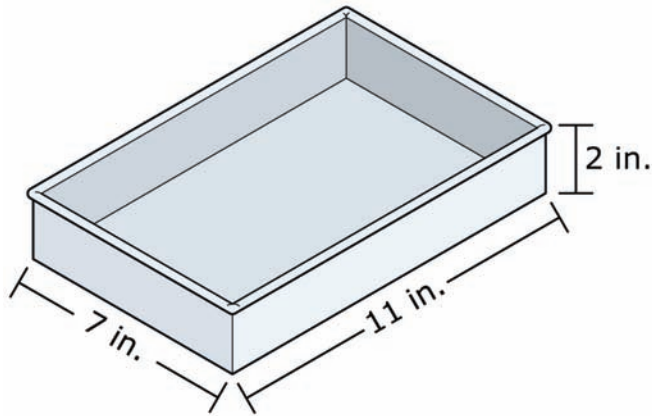
**1 point:** The student demonstrates some understanding of how to solve problems by using the structure of an expression to find ways to rewrite it. The student makes one or two minor errors in computation, such as combining a set of terms incorrectly when completing the square.

**0 points:** The student demonstrates inconsistent understanding of how to solve problems by using the structure of an expression to find ways to rewrite it. The student makes little or no use of knowledge or problem-solving strategies and does not attempt to complete the square when rearranging the formula.

**MAT.HS.CR.2.ONUMQ.A.603**

|  |  |
|--|--|
| Sample Item ID:  | MAT.HS.CR.2.ONUMQ.A.603  |
| Grade:   | HS   |
| Primary Claim:   | <b>Claim 2: Problem Solving</b><br>Students can solve a range of complex, well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.  |
| Secondary Claim(s):                                      | Claim 1: Concepts and Procedures<br>Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.   |
| Primary Content Domain:                                  | Number and Quantity  |
| Secondary Content Domain(s):                             |  |
| Assessment Target(s):                                    | 2 A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.<br><br>2 D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).<br><br>1 C: Reason quantitatively and use units to solve problems. |
| Standard(s):   | N-Q.1  |
| Mathematical Practice(s):                                | 1, 2, 4  |
| DOK:   | 2  |
| Item Type:   | CR   |
| Score Points:  | 1  |
| Difficulty:  | H  |
| Key:   | $1\frac{1}{8}$   |
| Stimulus/Source:   |  |
| Target-specific attributes (e.g., accessibility issues): |  |
| Notes:   | The response box accepts up to 5 numeric characters, plus a fraction bar (/) and decimal point (.)   |

Hannah makes 6 cups of cake batter. She pours and levels all the batter into a rectangular cake pan with a length of 11 inches, a width of 7 inches, and a depth of 2 inches.



One cubic inch is approximately equal to 0.069 cup.

What is the depth of the batter in the pan when it is completely poured in? Round your answer to the nearest  $\frac{1}{8}$  of an inch.

Key:

Correct responses to this item will receive 1 point.

1 point: For correct answer  $1\frac{1}{8}$  or 1.125 inches.



**MAT.HS.CR.2.OSTCP.D.070**

|  |   |
|--|---|
| Sample Item ID:  | MAT.HS.CR.2.OSTCP.D.070   |
| Grade:   | HS  |
| Primary Claim:   | <b>Claim 2: Problem Solving</b><br>Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.  |
| Secondary Claim(s):                                      | Claim 1: Concepts and Procedures<br>Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.  |
| Primary Content Domain:                                  | Statistics and Probability  |
| Secondary Content Domain(s):                             |   |
| Assessment Target(s):                                    | 2 D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).<br><br>2 A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace. |
| Standard(s):   | S-CP.4  |
| Mathematical Practice(s):                                | 1, 2, 7   |
| DOK:   | 2   |
| Item Type:   | CR  |
| Score Points:  | 2   |
| Difficulty:  | M   |
| Key:   | Part A: 0.4 (or equivalent); Part B: 0.2 (or equivalent)  |
| Stimulus/Source:   |   |
| Target-specific attributes (e.g., accessibility issues): |   |
| Notes:   |   |

Jaime randomly surveyed some students at his school to see what they thought of a possible increase to the length of the school day. The results of his survey are shown in the table below.

**Lengthening School Day Survey**

| <b>Grade</b> | <b>In Favor</b> | <b>Opposed</b> | <b>Undecided</b> |
|--------------|-----------------|----------------|------------------|
| 9            | 12              | 6              | 9                |
| 10           | 15              | 3              | 11               |
| 11           | 8               | 12             | 10               |
| 12           | 5               | 16             | 9                |

**Part A**

A newspaper reporter will randomly select a Grade 11 student from this survey to interview. What is the probability that the student selected is opposed to lengthening the school day?

**Part B**

The newspaper reporter would also like to interview a student in favor of lengthening the school day. If a student in favor is randomly selected, what is the probability that this student is also from Grade 11?

*Key:*

Each item is scored independently and will receive 1 point.

**Part A** 0.4 (or equivalent)

**Part B** 0.2 (or equivalent)

**MAT.HS.CR.2.OAREI.A.032**

|  |  |
|--|--|
| Sample Item ID:  | MAT.HS.CR.2.OAREI.A.032  |
| Grade:   | HS   |
| Primary Claim:   | <b>Claim 2: Problem Solving</b><br>Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies. |
| Secondary Claim(s):                                      | Claim 1: Concepts and Procedures<br>Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.                 |
| Primary Content Domain:                                  | Algebra  |
| Secondary Content Domain(s):                             |  |
| Assessment Target(s):                                    | 2 A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.<br><br>2 C: Interpret results in the context of a situation.                   |
| Standard(s):   | A-REI.6  |
| Mathematical Practice(s):                                | 1, 7   |
| DOK:   | 2  |
| Item Type:   | CR   |
| Score Points:  | 1  |
| Difficulty:  | M  |
| Key:   | vegetarian: \$7 and chicken: \$8   |
| Stimulus/Source:   |  |
| Target-specific attributes (e.g., accessibility issues): |  |
| Notes:   |  |

A restaurant serves a vegetarian and a chicken lunch special each day. Each vegetarian special is the same price. Each chicken special is the same price. However, the price of the vegetarian special is different from the price of the chicken special.

- On Thursday, the restaurant collected \$467 selling 21 vegetarian specials and 40 chicken specials.
- On Friday, the restaurant collected \$484 selling 28 vegetarian specials and 36 chicken specials.

What is the cost of each lunch special?

Vegetarian: \_\_\_\_\_

Chicken: \_\_\_\_\_

*Key:*

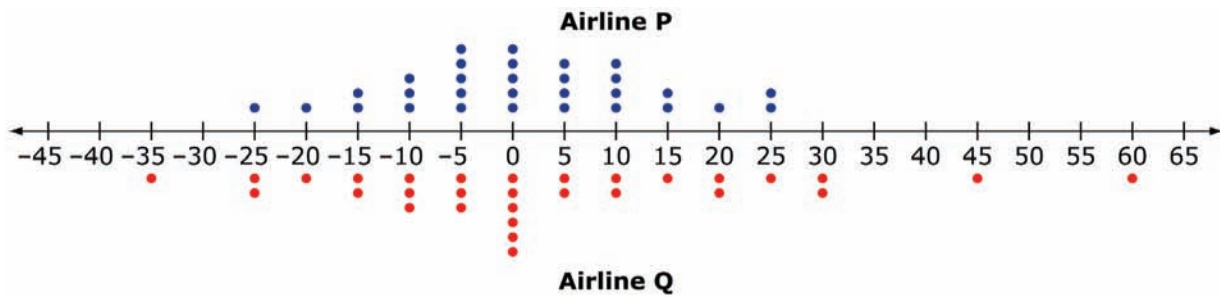
Correct responses to this item will receive 1 point.

1 point: vegetarian: \$7 and chicken: \$8

**MAT.HS.ER.2.00SID.C.264**

|  |  |
|--|--|
| Sample Item ID:  | MAT.HS.ER.2.00SID.C.264  |
| Grade:   | HS   |
| Primary Claim:   | <b>Claim 2: Problem Solving</b><br>Students can solve a range of complex, well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.  |
| Secondary Claim(s):                                      | Claim 1: Concepts and Procedures<br>Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.   |
| Primary Content Domain:                                  | Statistics and Probability   |
| Secondary Content Domain(s):                             |  |
| Assessment Target(s):                                    | 2C: Interpret results in the context of a situation.<br><br>2A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.<br><br>2B: Select and use appropriate tools strategically.<br><br>1P: Summarize, represent, and interpret data on a single count or measurement variable. |
| Standard(s):   | S-ID.3   |
| Mathematical Practice(s):                                | 1, 5   |
| DOK:   | 2  |
| Item Type:   | ER   |
| Score Points:  | 2  |
| Difficulty:  | M  |
| Key:   | See Sample Top-Scoring Response.   |
| Stimulus/Source:   |  |
| Target-specific attributes (e.g., accessibility issues): |  |
| Notes:   |  |

The dot plots below compare the number of minutes 30 flights made by two airlines arrived before or after their scheduled arrival times.



- Negative numbers represent the minutes the flight arrived **before** its scheduled time.
- Positive numbers represent the minutes the flight arrived **after** its scheduled time.
- Zero indicates the flight arrived **at** its scheduled time.

Based on these data, from which airline will you choose to buy your ticket? Use the ideas of center and spread to justify your choice.

*Sample Top-Score Response:*

I would choose to buy the ticket from Airline P. Both airlines are likely to have an on-time arrival since they both have median values at 0. However, Airline Q has a much greater range in arrival times. Airline Q could arrive anywhere from 35 minutes early to 60 minutes late. For Airline P, this flight arrived within 10 minutes on either side of the scheduled arrival time about 2/3 of the time, and for Airline Q, that number was only about 1/2. For these reasons, I think Airline P is the better choice.

*Scoring Rubric:*

Responses to this item will receive 0–2 points, based on the following:

**2 points:** The student has a solid understanding of how to make productive use of knowledge and problem-solving skills by comparing center and spread of two data sets using a graph and interpreting the results. The student chooses Airline P and clearly explains that both airlines have the same center but that Airline P has a smaller spread.

**1 point:** The student has some understanding of how to make productive use of knowledge and problem-solving skills by comparing center and spread of two data sets using a graph and interpreting the results. The student states that either airline could be chosen because they have the same median and does not address the issue of spread.

**OR** The student states that both airlines have the same median and chooses Airline P but does not justify the choice based on spread. **OR** The student explains that Airline P would be the better choice based on the smaller spread but does not identify that both airlines have the same median.

**0 points:** The student demonstrates an inconsistent understanding of how to make productive use of knowledge and problem-solving skills by comparing center and spread of two data sets using a graph and interpreting the results. The student does not state that the two airlines have the same median and that Airline Q has greater spread. The student either does not make a choice between the two airlines or makes a choice but does not defend it using center or variation.



**MAT.HS.ER.3.00NRN.B.085**

|  |  |
|--|--|
| Sample Item ID:  | MAT.HS.ER.3.00NRN.B.085  |
| Grade:   | HS   |
| Primary Claim:   | <b>Claim 3: Communicating Reasoning</b><br>Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.   |
| Secondary Claim(s):                                      | Claim 1: Concepts and Procedures<br>Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.   |
| Primary Content Domain:                                  | Number and Quantity  |
| Secondary Content Domain(s):                             |  |
| Assessment Target(s):                                    | 3 B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.<br><br>3 C: State logical assumptions being used.<br><br>3 F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions.<br><br>1 B: Use properties of rational and irrational numbers. |
| Standard(s):   | N-RN.3   |
| Mathematical Practice(s):                                | 1, 3, 6  |
| DOK:   | 3  |
| Item Type:   | ER   |
| Score Points:  | 3  |
| Difficulty:  | M  |
| Key:   | See Sample Top-Score Response.   |
| Stimulus/Source:   |  |
| Target-specific attributes (e.g., accessibility issues): |  |
| Notes:   | Part of PT set   |

**Part A**

The rectangle shown below has a length of 6 feet.



6 feet

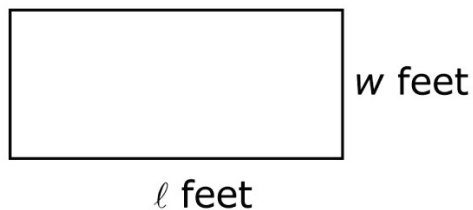
The value of the area of the rectangle, in square feet, is an irrational number. Therefore, the number that represents the width of the rectangle must be —

- Ⓐ a whole number.
- Ⓑ a rational number.
- Ⓒ an irrational number.
- Ⓓ a non-real complex number.

[Note: After **Part A** is completed, then the student must not be allowed to go back to it once they have moved to **Part B**.]

### **Part B**

The length,  $\ell$ , and width,  $w$ , of the rectangle shown below have values that are rational numbers.



Construct an informal proof that shows that the value of the area, in square feet, of the rectangle must be a rational number.

*Sample Top-Score Response:***Part A C****Part B**

Given:  $l$  is rational;  $w$  is rational.

Prove:  $l \times w$  is rational.

Proof: Since  $l$  is rational, by the definition of rational number,  $l$  can be written in the form  $\frac{a}{b}$ , where  $a$  and  $b$  are both integers and  $b$  is nonzero. Similarly, since  $w$  is rational, by the definition of rational number,  $w$  can be written in the form  $\frac{c}{d}$ , where  $c$  and  $d$  are both integers and  $d$  is nonzero. Then  $l \times w = \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ .

Since the set of integers is closed under the operation of multiplication, both  $ac$  and  $bd$  are integers. Thus  $l \times w$  is the ratio of two integers. So by the definition of rational number,  $l \times w$  is rational.

*Scoring Rubric:*

**Part A** 1 point for selecting the correct response of C  
0 points for selecting any response other than C

- A The student thinks that since the length is a whole number, so is the width.
- B The student confuses what type of factors produce a product that is irrational.
- C Key
- D The student does not have a clear understanding of what type of factors produce a product that is irrational.

**Part B**

Responses to *Part B* of this item will receive 0-2 points, based on the following:

**2 points:** The student has a solid understanding of how to clearly and precisely construct a viable argument to support their own reasoning for proving that the product of two rational numbers is a rational number. The student clearly communicates the given information and what is to be proved. The student clearly constructs a logical sequence of steps, with reasons, to prove that the area  $A$  is rational.

**1 point:** The student has some understanding of how to clearly and precisely construct a viable argument to support their own reasoning for proving that the product of two rational numbers is a rational number. The student communicates the given information and what is to be proved, but demonstrates some flawed or incomplete reasoning when constructing a logical sequence of steps, with reasons, to prove that the area  $A$  is a rational number.

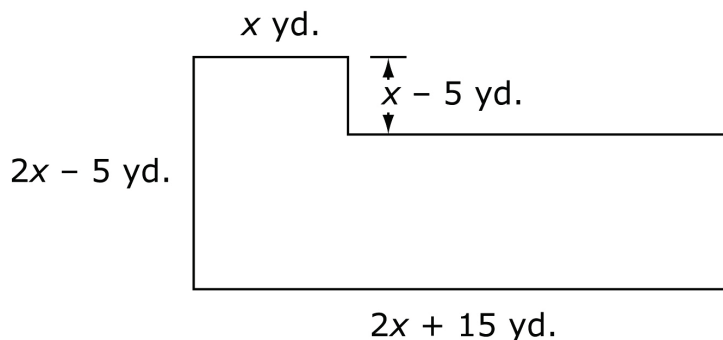
**0 points:** The student demonstrates inconsistent understanding of how to clearly and precisely construct a viable argument to support their own reasoning for proving that the product of two rational numbers is a rational number. The student does not clearly communicate or fails to communicate the given information or what is to be proved, and demonstrates greatly flawed or incomplete reasoning when trying to construct a logical sequence of steps, with reasons, to prove that the area  $A$  is a rational number.

**MAT.HS.ER.3.OAAPR.F.045**

|  |   |
|--|---|
| Sample Item ID:  | MAT.HS.ER.3.OAAPR.F.045   |
| Grade:   | HS  |
| Claim:   | <b>Claim 3. Communicating Reasoning</b><br>Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.  |
| Secondary Claim(s):                                      | Claim 1: Concepts and Procedures<br>Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.  |
| Content Domain:  | Algebra   |
| Assessment Target(s):                                    | 3 F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions.<br><br>3 B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.<br><br>1 F: Perform arithmetic operations on polynomials. |
| Standard(s):   | A-APR.1   |
| Mathematical Practice(s):                                | 1, 2, 3, 6  |
| DOK:   | 3   |
| Item Type:   | ER  |
| Score Points:  | 3   |
| Difficulty:  | H   |
| Key:   | See Sample Top-Score Response.  |
| Stimulus/Source:   |   |
| Target-specific attributes (e.g., accessibility issues): |   |
| Notes:   | Multi-part item – Part of PT set  |

**Part A**

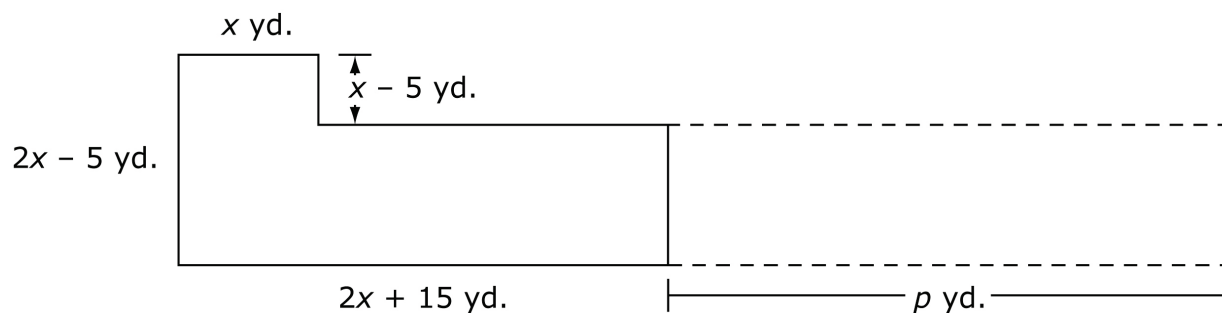
A town council plans to build a public parking lot. The outline below represents the proposed shape of the parking lot.



Write an expression for the area, in square feet, of this proposed parking lot. Explain the reasoning you used to find the expression.

**Part B**

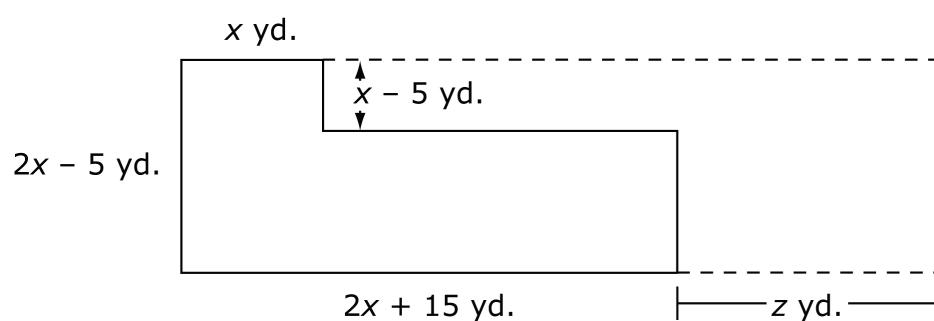
The town council has plans to double the area of the parking lot in a few years. They create two plans to do this. The first plan increases the length of the base of the parking lot by  $p$  yards, as shown in the diagram below.



Write an expression in terms of  $x$  to represent the value of  $p$ , in feet. Explain the reasoning you used to find the value of  $p$ .

**Part C**

The town council's second plan to double the area changes the shape of the parking lot to a rectangle, as shown in the diagram below.



Can the value of  $z$  be represented as a polynomial with integer coefficients? Justify your reasoning.

*Sample Top-Score Response:*

**Part A**

Missing vertical dimension is  $2x - 5 - (x - 5) = x$ .

$$\begin{aligned} \text{Area} &= x(x - 5) + x(2x + 15) \\ &= x^2 - 5x + 2x^2 + 15x \\ &= 3x^2 + 10x \text{ square yards} \end{aligned}$$

**Part B**

Doubled area =  $6x^2 + 20x$  square yards.

Area of top left corner =  $x^2 - 5x$  square yards.

$$\begin{aligned} \text{Area of lower portion with doubled area} &= 6x^2 + 20x - (x^2 - 5x) \\ &= 5x^2 + 25x \text{ square yards} \end{aligned}$$

Since the width remains  $x$  yards, the longest length must be

$$(5x^2 + 25x) \div x = 5x + 25 \text{ yards long.}$$

$$\text{So, } y = 5x + 25 - (2x + 15) = 5x + 25 - 2x - 15 = 3x + 10 \text{ yards.}$$

**Part C**

If  $z$  is a polynomial with integer coefficients, the length of the rectangle,  $2x + 15 + z$ , would be a factor of the doubled area. Likewise,  $2x - 5$  would be a factor of the doubled area. But  $2x - 5$  is not a factor of  $6x^2 + 20x$ . So  $2x + 15 + z$  is not a factor either. Therefore,  $z$  cannot be represented as a polynomial with integer coefficients.

**Scoring Rubric:**

*Responses to this item will receive 0-3 points, based on the following:*

**3 points:** The student has a solid understanding of how to articulate reasoning with viable arguments associated with adding, subtracting, and multiplying polynomials. The student answers *parts A* and *B* correctly, showing all relevant work or reasoning. The student also clearly explains assumptions made in *Part C* as well as showing how they lead to a refutation of the conjecture that a given polynomial has integer coefficients.

**2 points:** The student understands how to add, subtract, and multiply polynomials but cannot clearly articulate reasoning with viable arguments associated with these tasks. The student answers *parts A* and *B* correctly, showing all relevant work or reasoning. However, the student has flawed or incomplete reasoning associated with assumptions made in *Part C* that lead to a refutation of the conjecture that a given polynomial has integer coefficients.

**1 point:** The student has only a basic understanding of how to articulate reasoning with viable arguments associated with adding, subtracting, and multiplying polynomials. The student makes one or two computational errors in *parts A* and *B*. The student also has flawed or incomplete reasoning associated with assumptions made in *Part C* that lead to a refutation of the conjecture that a given polynomial has integer coefficients.

**0 points:** The student demonstrates inconsistent understanding of how to articulate reasoning with viable arguments associated with adding, subtracting, and multiplying polynomials. The student makes three or more computational errors in *parts A* and *B*. The student also has missing or flawed reasoning related to determining whether a given polynomial has integer coefficients.



HS Mathematics Sample ER Item Claim 4

**MAT.HS.ER.4.00FLE.E.566**

|  |   |
|--|---|
| Sample Item ID:  | MAT.HS.ER.4.00FLE.E.566   |
| Grade:   | HS  |
| Primary Claim:   | <b>Claim 4: Modeling and Data Analysis</b><br>Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.   |
| Secondary Claim(s):                                      | Claim 3: Communicating Reasoning<br>Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.<br><br>Claim 1: Concepts and Procedures<br>Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.   |
| Primary Content Domain:                                  | Functions   |
| Secondary Content Domain(s):                             |   |
| Assessment Target(s):                                    | 4 E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.<br><br>3 E: Distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in the argument—explain what it is.<br><br>4 A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.<br><br>3 B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.<br><br>3 C: State logical assumptions being used.<br><br>4 D: Interpret results in the context of a situation.<br><br>3 F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions. |
| Standard(s):   | F-LE.1, F-LE.3, F-LE.5  |
| Mathematical Practice(s):                                | 1, 3, 4, 6, 7, 8  |
| DOK:   | 4   |
| Item Type:   | ER  |
| Score Points:  | 4   |
| Difficulty:  | M   |
| Key:   | See Sample Top-Score Response.  |
| Stimulus/Source:   |   |
| Target-specific attributes (e.g., accessibility issues): |   |
| Notes:   | Part of PT set  |



**Part B**

Create a table of values to compare the expected salary increases for an employee with a starting salary of \$100,000 based on Mr. Miller's incorrect reasoning and the more reasonable expected salary increases. List these ranges in separate columns of the table up to the employee's sixth year at this company.

*Sample Top-Score Response:*

**Part A**

1. \$47,200 – \$58,000

2. Mr. Miller's reasoning is incorrect because he is treating the range of percent increases linearly instead of exponentially. He calculates each following year's increase range by

adding the amount calculated based on his first year's salary. What he should do is add the increase ranges from the first year to the first year's salary to find the range for his second year's salary. Then, he should multiply the higher second year's salary range by the range in percents and add those increase amounts to find the following year's amounts. Each following year's percent increases should be based off the prior year's increased salary ranges.

**Part B**

**Expected Salary Increases  
for \$100,000 Starting Salary**

| Year | Based on Incorrect Reasoning |                 | More Reasonable Increases |                 |
|------|------------------------------|-----------------|---------------------------|-----------------|
|      | Low Range (2%)               | High Range (5%) | Low Range (2%)            | High Range (5%) |
| 1    | \$100,000.00                 | \$100,000.00    | \$100,000.00              | \$100,000.00    |
| 2    | \$102,000.00                 | \$105,000.00    | \$102,000.00              | \$105,000.00    |
| 3    | \$104,000.00                 | \$110,000.00    | \$104,040.00              | \$110,250.00    |
| 4    | \$106,000.00                 | \$115,000.00    | \$106,120.80              | \$115,762.50    |
| 5    | \$108,000.00                 | \$120,000.00    | \$108,243.22              | \$121,550.63    |
| 6    | \$110,000.00                 | \$125,000.00    | \$110,408.08              | \$127,628.16    |

*Scoring Rubric:*

Responses to **Part A** of this item will receive 0–2 points, based on the following:

**2 points:** The student demonstrates a solid understanding of how to analyze complex, real-world scenarios to interpret and critique the reasoning of others. The student identifies the correct salary range for year ten and provides an accurate and complete critique of why the given reasoning for calculating the salary range is flawed.

**1 point:** The student has a limited understanding of how to analyze complex, real-world scenarios to interpret and critique the reasoning of others. The student identifies the correct salary range for year ten but provides a partially accurate critique of why the given reasoning for calculating the salary range is flawed. **OR** The student miscalculates the salary range for year ten but provides an accurate and complete critique of why the given reasoning for calculating the salary range is flawed.

**0 points:** The student demonstrates inconsistent understanding of how to analyze complex, real-world scenarios to interpret and critique the reasoning of others. The student does not determine the correct salary range for year ten and does not provide an accurate and complete critique of why the given reasoning for calculating the salary range is flawed.

Responses to **Part B** of this item will receive 0–2 points, based on the following:

**2 points:** The student demonstrates a solid understanding of how to construct mathematical models to make improvements to an existing model. The student provides a fully accurate table for each calculation.

## HS Mathematics Sample ER Item Claim 4

**1 point:** The student has some understanding of how to construct mathematical models to make improvements to an existing model. The student has a general understanding of which formulas are used to make the calculations in each column of the table but makes some minor calculation errors.

**0 points:** The student demonstrates inconsistent understanding of how to construct mathematical models to make improvements to an existing model. The student does not provide an accurate table for each calculation.

**MAT.HS.PT.4.OCORN.A.412**

|                              |   |
|------------------------------|---|
| Sample Item ID:              | MAT.HS.PT.4.OCORN.A.412   |
| Title:                       | Corn  |
| Grade:                       | HS  |
| Primary Claim:               | <b>Claim 4: Modeling and Data Analysis</b><br>Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.   |
| Secondary Claim(s):          | <p>Claim 1: Concepts and Procedures<br/>Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.</p> <p>Claim 2: Problem Solving<br/>Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.</p> <p>Claim 3: Communicating Reasoning<br/>Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.</p>  |
| Primary Content Domain:      | Number and Quantity   |
| Secondary Content Domain(s): | Geometry, Statistics, Algebra   |
| Assessment Target(s):        | <p>4 A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.</p> <p>4 D: Interpret results in the context of a situation.</p> <p>4F: Identify important quantities in a practical situation and map their relationships (e.g. using diagrams, two-way tables, graphs, flowcharts, or formulas).</p> <p>1 C: Reason quantitatively and use units to solve problems.</p> <p>1 N: Build a function that models a relationship between two quantities.</p> <p>1 P: Summarize, represent, and interpret data on a single count or measurement variable.</p> <p>2 A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.</p> <p>2 B: Select and use appropriate tools strategically.</p> <p>2 C: Interpret results in the context of a situation.</p> |
| Standard(s):                 | N-Q.1,N-Q.2, S-ID.6, G-MG.3, G-SRT.8, 7.RP.2, 7.EE.3, A-CED.1, 7.RP.1   |

HS Mathematics Sample PT Form Claim 4



|   |   |
|---|---|
| Mathematical Practice(s):   | 1, 2, 3, 4, 5, 6  |
| DOK:  | 4   |
| Item Type:  | PT  |
| Score Points:   | 20  |
| Difficulty:   | H   |
| How this task addresses the “sufficient evidence” for this claim: | The student uses concept of number and quantity, geometry, and statistics to provide rationale for the recommendation made regarding on- or off-site storage of harvested corn. The work is supported by the calculations.  |
| Target-specific attributes (e.g., accessibility issues):          | Accommodations may be necessary for students who have challenges with language processing, vision, or fine motor skills.  |
| Stimulus/Source:  | <a href="http://www.extension.iastate.edu/agdm/crops/html/a2-35.html">http://www.extension.iastate.edu/agdm/crops/html/a2-35.html</a><br><a href="http://www.extension.iastate.edu/agdm/wholefarm/pdf/c2-24.pdf">http://www.extension.iastate.edu/agdm/wholefarm/pdf/c2-24.pdf</a><br><a href="http://www.ksre.ksu.edu/library/agec2/mf2474.pdf">http://www.ksre.ksu.edu/library/agec2/mf2474.pdf</a><br><a href="http://www.michigan.gov/documents/Vol1-27GrainBinsandTanks_120836_7.pdf">http://www.michigan.gov/documents/Vol1-27GrainBinsandTanks_120836_7.pdf</a><br><a href="http://www.ces.purdue.edu/extmedia/gq/gq-3.html">http://www.ces.purdue.edu/extmedia/gq/gq-3.html</a> |
| Notes:  |   |
| Task Overview:  | Students will research the price of corn and the costs associated with rental storage and grain bin storage, as well as the costs associated with drying corn to remove moisture so that it can be stored. A recommendation will then be made based on this analysis as to what type of corn storage method a farmer should use.  |
| Teacher preparation / Resource requirements:                      | Resource requirements: Video access. Up to two days prior to the administration of this task, the teacher will provide class time to watch pretask videos. They may be watched as a class or individually. The teacher will also require students to perform a “prework” task in which they will research current prices of corn and liquid propane gas.  |
| Teacher Responsibilities During Administration:                   | After the prework, the teacher will find the average of the prices for corn and gas that have been submitted by the students. These averages will be the numbers used in Session 1. The teacher should check for feasibility.<br><br>During Session 1, the students will record values of certain quantities on a note sheet that will be needed for work during Session 2. After Session 1, the teacher will collect the note sheets from the students and return them to the students the following day. The students will need these responses to continue work on the second day.<br><br>Monitor individual student work as necessary.  |
| Time Requirements:  | Excluding the prework, the task will be completed in two 60-minutes sessions. Parts A through C will be completed during Session 1 and Parts D and E will be completed during Session 2.  |

## HS Mathematics Sample PT Form Claim 4

### Prework:

Students will watch two short videos describing the harvesting and storing of corn for market. These videos will assist students, especially those unfamiliar with the work on a farm, by giving them a snapshot of this process. They may also supplement the reading load of these tasks for ELLs.

Here are some examples of ones that might be used:

- <http://www.youtube.com/watch?v=1jhuNDuLaps>
- <http://www.youtube.com/watch?v=iddFy6A9uHg>

Students will also be asked to research the current cost of corn and of LPG (liquid propane gas).



### Your Assignment:

In this task you will assume the role of consultant for a farmer. You will analyze the options available to the farmer for handling the storage of shelled field corn (shown in the pictures above). In the past, the farmer has sold the corn as it was harvested, and did not store the corn to be sold in the future. The farmer has increased the number of acres used to grow corn, and now is exploring the cost of storing the corn until after the harvest is complete and then selling it. You will analyze two storage options available to the farmer for storing the grain that is harvested.

- The corn can be stored in grain bins constructed on the farm.
- The corn can be stored in rental storage close to the farm.

Following the tasks, you will recommend which type of storage the farmer should use.

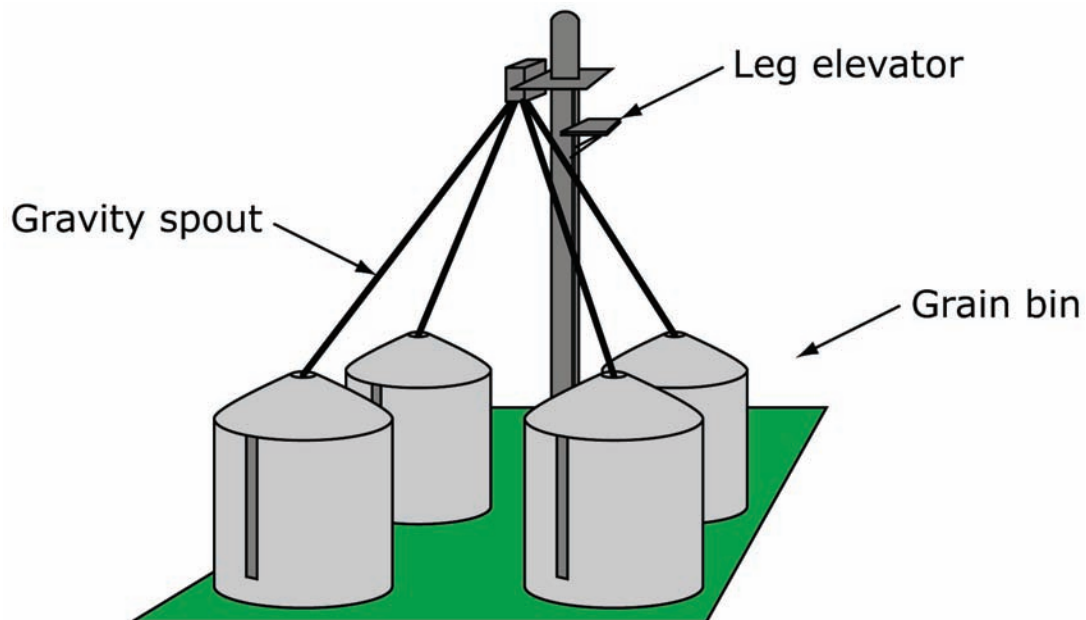


## Corn Storage

### Session 1

#### Part A

Your first job is to determine the most efficient cost for constructing 4 grain bins which can be used to store the harvested corn. A leg elevator, which moves the corn from ground level into the bins, must also be built. The bins must be able to hold the 132,000 bushels of corn from the harvest. Each bin should include a ventilated floor, fan and heat. A cost table for building various size bins is shown below.



| Cost of Grain Bins |               |                    |                         |                     |                  |                       |                   |
|--------------------|---------------|--------------------|-------------------------|---------------------|------------------|-----------------------|-------------------|
| Diameter (feet)    | Height (feet) | Capacity (bushels) | Cost Without Floor (\$) | Add for             |                  |                       |                   |
|                    |               |                    |                         | Concrete Floor (\$) | Steel Floor (\$) | Ventilated Floor (\$) | Fan and Heat (\$) |
| 30                 | 19            | 10,775             | 11,525                  | 1,650               | 1,130            | 4,250                 | 2,950             |
|                    | 24            | 13,625             | 16,000                  | 1,775               | 1,130            | 4,250                 | 2,950             |
|                    | 32            | 18,175             | 20,100                  | 1,975               | 1,130            | 4,250                 | 2,950             |
| 33                 | 24            | 16,475             | 17,725                  | 2,050               | 1,320            | 5,100                 | 3,025             |
|                    | 27            | 18,550             | 20,050                  | 2,100               | 1,320            | 5,100                 | 3,025             |
|                    | 32            | 21,975             | 24,950                  | 2,550               | 1,320            | 5,100                 | 3,025             |
| 36                 | 24            | 19,625             | 21,575                  | 2,575               | 1,540            | 6,000                 | 3,075             |
|                    | 27            | 22,075             | 23,475                  | 2,675               | 1,540            | 6,000                 | 3,075             |
|                    | 32            | 26,150             | 26,100                  | 2,775               | 1,540            | 6,000                 | 3,075             |
|                    | 40            | 32,700             | 28,925                  | 2,950               | 1,540            | 6,000                 | 3,075             |
| 42                 | 27            | 30,050             | 28,450                  | 3,650               | 2,065            | 8,100                 | 3,225             |
|                    | 32            | 35,600             | 32,525                  | 3,875               | 2,065            | 8,100                 | 3,225             |
|                    | 40            | 44,500             | 39,650                  | 4,075               | 2,065            | 8,100                 | 3,225             |
|                    | 48            | 53,425             | 47,200                  | 4,400               | 2,065            | 8,100                 | 3,225             |
| 48                 | 27            | 39,250             | 41,150                  | 4,775               | 2,640            | 10,450                | 3,350             |
|                    | 32            | 46,500             | 48,900                  | 5,050               | 2,640            | 10,450                | 3,350             |
|                    | 40            | 58,150             | 55,000                  | 5,300               | 2,640            | 10,450                | 3,350             |
|                    | 48            | 69,775             | 61,650                  | 5,750               | 2,640            | 10,450                | 3,350             |

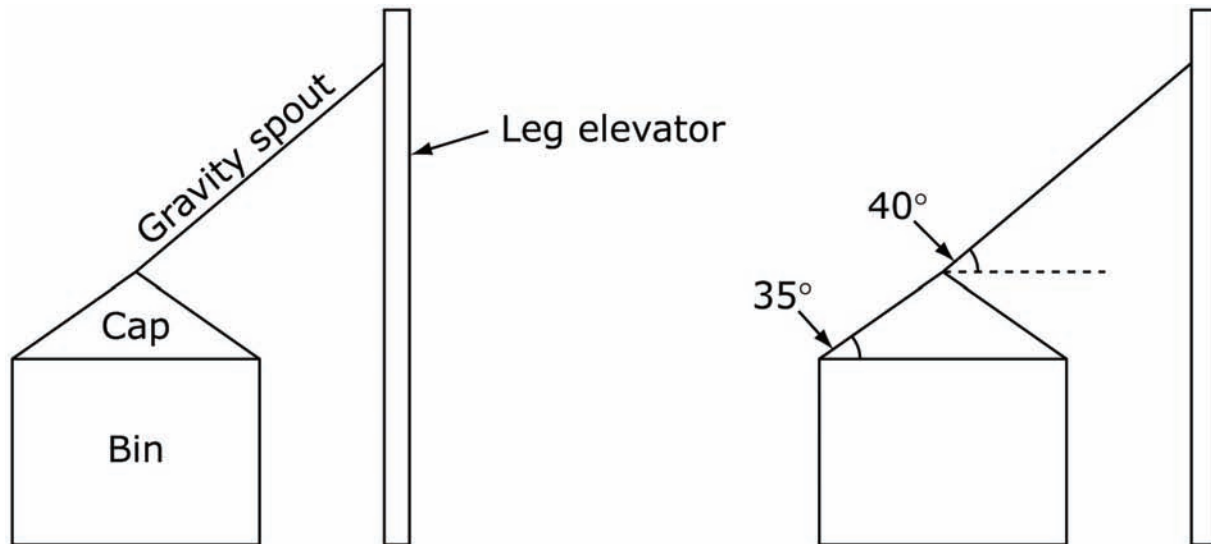
All 4 bins must have the same capacity.

The bins must be built to the following specifications.

- The height listed in the table does not include the height of the conical cap on top of the bin. The cap forms a 35° angle with the base.
- The distance from the outer edge of the bins to the leg elevator will be 15 feet.
- A gravity spout is placed so that it runs from the top of the cap to a point that is 2 feet below the top of the elevator leg. To account for certain moisture content the gravity

spouts will slope 40 degrees to the horizontal.

- The average cost involved in the construction of the leg elevator is \$15,000 plus \$125 for every foot in height.
- The gravity spouts cost \$20 per foot.



Find the most efficient cost of the construction. Be sure to include the bins (caps are included in the price), gravity spouts, and leg elevator.

### Part B

When corn is harvested, the moisture content varies, but is typically above the level desired for selling or storing corn, so the corn must be dried. The moisture content is given as a percent that represents the proportion of the weight of the corn that is from water. For example, if you had 40 lbs of corn at 25% moisture content, it would consist of 10 lbs of water and 30 lbs of dry material. As corn dries, the amount of water decreases, but the amount of dry material stays the same, so the percent of weight from water will decrease.

The final moisture contents for various purposes are as follows:

- 15.5% to sell at market
- 14.0% to store at a rental storage facility
- 13.5% to store in grain bins constructed on the farm

There is a cost for drying corn to 15.5% moisture to be able to sell it at market, but there is extra cost to dry it below 15.5%. This extra cost is a cost of storage since it must be paid only if the grain is to be stored and not sold at market.

Assuming corn is harvested at an initial moisture content of 20% and you use LP gas as fuel for your dryer, use the information in tables 1 and 2 below to calculate the extra cost per bushel of drying corn to a final moisture content of 14% and 13.5%. Justify your answer mathematically and show all the steps in your calculation. You can use the regression tool in the spreadsheet provided if necessary. The BTUs required to dry corn to a final moisture content of 15.5% and 13.5% are not in the table but can be found using the provided regression tool.

### Energy (BTU's) Required to Dry a Bushel of Wet Corn

| Final Moisture Content | Initial Moisture Content |        |        |        |        |        |
|------------------------|--------------------------|--------|--------|--------|--------|--------|
|                        | 20%                      | 22%    | 24%    | 26%    | 28%    | 30%    |
| 17%                    | 5,625                    | 8,744  | 11,714 | 14,487 | 17,086 | 19,545 |
| 16%                    | 7,522                    | 10,596 | 13,506 | 16,241 | 18,784 | 21,188 |
| 15%                    | 9,579                    | 12,589 | 15,447 | 18,118 | 20,624 | 22,978 |
| 14%                    | 11,635                   | 14,582 | 17,388 | 19,994 | 22,463 | 24,768 |
| 13%                    | 13,878                   | 16,774 | 19,528 | 22,088 | 24,486 | 26,744 |

### Energy Content (BTU's) per Unit of Fuel

| Fuel Type   | Unit       | BTU's per Unit of Fuel |
|-------------|------------|------------------------|
| Oil         | Gallon     | 140,000                |
| LP gas      | Gallon     | 92,000                 |
| Electricity | kWh        | 3,414                  |
| Natural gas | Cubic foot | 1,000                  |

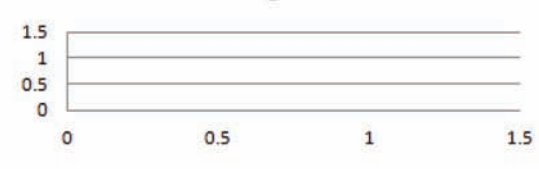
To use the regression tool below, enter labels for the axes and pairs of independent and dependent variable values in the spreadsheet.

Regression Tool:


|                   | Independent Variable | Dependent Variable |  |  |
|-------------------|----------------------|--------------------|--|--|
| Enter axis labels |                      |                    |  |  |
|                   |                      |                    |  |  |
| Enter Quantities  |                      |                    |  |  |
|                   |                      |                    |  |  |
|                   |                      |                    |  |  |
|                   |                      |                    |  |  |

**Linear Regression**

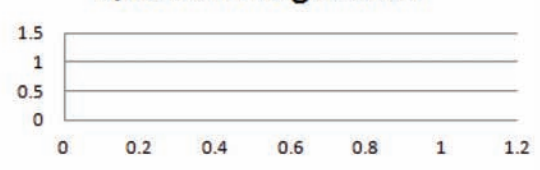


**Exponential Regression**



**Quadratic Regression**



Enter your final answers:

Extra cost to dry 1 bushel of corn to 14% = \_\_\_\_\_

Extra cost to dry 1 bushel of corn to 13.5% = \_\_\_\_\_

(Record these values on your note sheet; you will need them in a later part.)

### **Part C**

Corn is composed of dry material and water (moisture). For example, corn at 16% moisture would be composed of 84% dry material. At 15.5% moisture, one bushel of corn weighs 56 pounds. Complete the table below to show the amounts of dry material for 56 pounds of corn at each of the moisture levels. Show all work to get the values.

| <b>Moisture Level</b> | <b>Amount of Dry Material (pounds)</b> |
|-----------------------|--|
| 15.5%                 |  |
| 14%                   |  |
| 13.5%                 |  |

Enter the average price per bushel of corn that you found during the prework in the blank below.

Corn at 15.5% moisture sells for \$\_\_\_\_\_ per bushel.

What is the price per pound for the dry material in 56 pounds of corn at 15.5% moisture? Show all work, and round your answer to the thousandths place.

At this rate of dollars per pound of dry material, what is the value, in dollars, of the dry material in 56 pounds of 14% corn

and in 56 pounds of 13.5% moisture corn? Show all work.

When 56 pounds of corn is sold at market, the buyer receives more dry material if the corn has lower moisture content. This means that there is a cost to the farmer of drying corn for storage, since each bushel sold will contain more dry material than it would have at higher moisture content. This cost is called the shrinkage cost.

For 56 pounds of corn sold:

Shrinkage Cost = value of dry material – selling price

Find the shrinkage costs when corn is sold at 14% moisture and at 13.5% moisture. Show all work.

Enter your final answers.

Shrinkage cost, per bushel, for selling corn at 14% = \_\_\_\_\_

Shrinkage cost, per bushel, for selling corn at 13.5% = \_\_\_\_\_

(Record these values on your note sheet; you will need them in a later part.)

## Session 2

### ***Part D***

In this part, you will calculate the total rental cost of storing 132,000 bushels of corn at a grain elevator close to the farm, which is called rental storage. The farmer provides you with the following information.

- In January, February, and August, 2 truckloads of corn can be transported to market each day to be sold.
- In March, April, May, June, and July, 1 truckload of corn can be transported to market each day to be sold.
- Each truck the farm uses for transporting corn holds 600 bushels of corn.
- On average, corn is transported to market 20 days each

month.

- The farmer only transports and sells grain beginning in January.
- The cost for storing grain is \$0.09 per bushel for 3 months and then \$0.02 per bushel for each additional month past 3 months.
- The monthly storage cost for corn stored past 3 months is calculated based on the amount of corn in rental storage at the beginning of the month.
- From past experience, the farmer estimates the following percentages of corn harvested each month.

| <b>Month</b> | <b>Percent of Crop Harvested and Put in Storage</b> |
|--------------|---|
| September    | 20%   |
| October      | 40%   |
| November     | 30%   |
| December     | 10%   |

Enter the necessary amounts in the provided spreadsheet to calculate the total rental cost of storing the corn at a grain elevator close to the farm. Amounts can only be entered in cells that are shaded yellow.



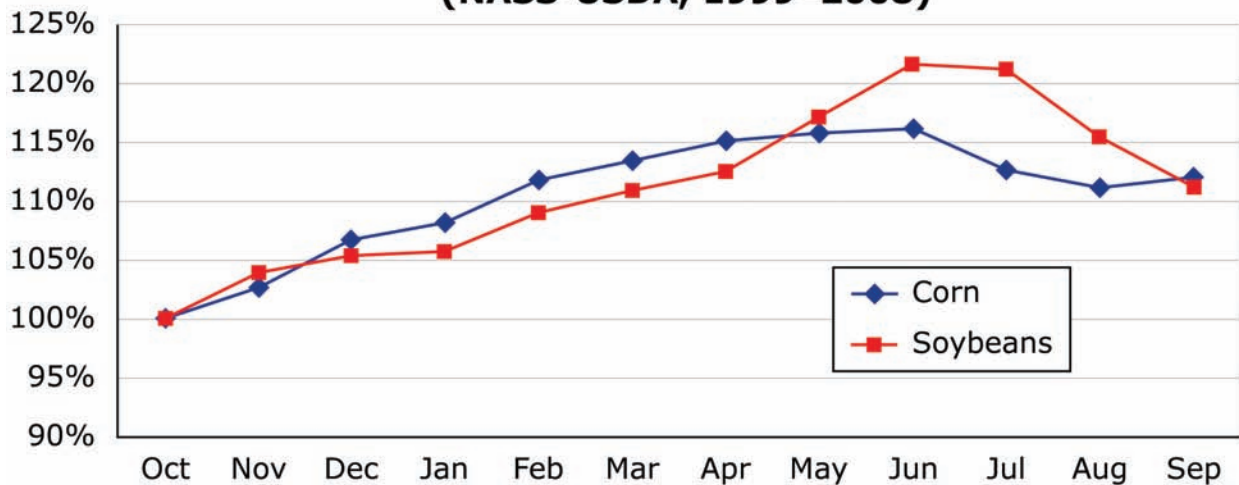
| Cost to store 1 bushel for 3 months=                             |                                |  |                                      |   |  |                        |
|--|--------------------------------|--|--------------------------------------|---|--|------------------------|
| Cost to store 1 bushel for each month past the initial 3 months= |                                |  |                                      |   |  |                        |
| Number of bushels =  |                                |  |                                      |   |  |                        |
| Month  | Percent of Crop Put in Storage | Number of Bushels That It Is Possible to Transport | Percent of Crop Removed From Storage | Number of Bushels Removed From Storage During Month | Number of Bushels Remaining In Storage at End of Month | Cost of Rental Storage |
| September  |                                |  |                                      |   |  | \$0.00                 |
| October  |                                |  |                                      |   |  | \$0.00                 |
| November   |                                |  |                                      |   |  | \$0.00                 |
| December   |                                |  |                                      |   |  | \$0.00                 |
| January  |                                |  | #DIV/0!                              | #DIV/0!   | #DIV/0!  | \$0.00                 |
| February   |                                |  | #DIV/0!                              | #DIV/0!   | #DIV/0!  | #DIV/0!                |
| March  |                                |  | #DIV/0!                              | #DIV/0!   | #DIV/0!  | #DIV/0!                |
| April  |                                |  | #DIV/0!                              | #DIV/0!   | #DIV/0!  | #DIV/0!                |
| May  |                                |  | #DIV/0!                              | #DIV/0!   | #DIV/0!  | #DIV/0!                |
| June   |                                |  | #DIV/0!                              | #DIV/0!   | #DIV/0!  | #DIV/0!                |
| July   |                                |  | #DIV/0!                              | #DIV/0!   | #DIV/0!  | #DIV/0!                |
| August   |                                |  | #DIV/0!                              | #DIV/0!   | #DIV/0!  | #DIV/0!                |
| Total Cost of Rental Storage=                                    |                                |  |                                      |   |  | #DIV/0!                |

**Part E**

In this part, you will analyze the cost of building grain bins to store corn on the farm. Based on the time series plot below, the farmer thinks that it might be more cost effective to build grain bins rather than paying for rental storage. Storing corn in grain

bins on the farm will enable the farmer to sell corn to many possible buyers at times during the year when the price of corn will be higher than it is at harvest time.

**Monthly Cash Prices as % of September–October Price  
(NASS-USDA, 1999–2008)**



Calculate the average increase in selling price (\$ per bushel) that the farmer receives by selling corn during the months of January through August rather than selling all of the corn at harvest. The average increase in selling price (\$ per bushel) is \$0.20 more for grain stored in the farmer’s bins than for grain stored in rental storage because rental storage charges a fee to remove grain to sell elsewhere.

Use the results of your calculations and any other necessary information to enter values in the spreadsheet below to calculate the cost for storing corn in grain bins and in rental storage. You will be provided with the note sheet on which you recorded the **current** cost per bushel of corn that you found in your prework, and the results of your calculations from previous parts.

**Grain Storage Investment Comparison**

**Ag Decision Maker -- Iowa State University Extension**

Place the cursor over cells with red triangles to read comments.

Enter your input values in yellow shaded cells.

**Storage Type**

Number of bushels stored  
 Total cost of bin construction (\$)  
 bin (including floor, fan, heat), spouts, leg elevator  
 additional augers, fans, conveyers, etc.  
 Useful life for bin (years)  
 Useful life for augers, fans, conveyers (years)  
 Interest rate for financing initial investment (%)

Average moisture at harvest (%)  
 Moisture level for storage (%)  
 Extra handling cost (\$ per bu.)  
 Extra transportation cost (\$ per bu.)  
 Quality loss for on-farm storage (%)  
 Fans, total horsepower  
 Hours fans will run, total  
 Electricity rate (\$ per KWH)  
 Add'l hours per month spent managing & handling grain  
 Labor value (\$ per hour)

Commercial storage charges  
 Minimum charge or base rate (\$ per bu.)  
 Base period (months)  
 Rate per month after base period (\$ per bu. per month)

Expected September-October cash selling price of corn (\$ per bu.)  
 Average increase in selling price (\$ per bu.)  
 Short-term interest rate (%)  
 Average length of storage (months)

| Corn               |                |
|--------------------|----------------|
| On-farm Investment | Rental Storage |
| 1,000 bu.          | 1,000 bu.      |
| \$ 1,000           |                |
| \$ 5,000           |                |
| 25 years           |                |
| 10 years           |                |
| 4.6%               |                |
| 20.0%              | 20.0%          |
| 13.5%              | 14.0%          |
| \$ 0.015           |                |
| \$ 0.050           |                |
| 1.0%               |                |
| 10 hp              |                |
| 1440 hrs           |                |
| \$ 0.1000          |                |
| 5.00               |                |
| \$ 10.00           |                |
|                    | \$0.2100       |
|                    | 3 months       |
|                    | \$0.0280       |
| \$ 6.00            | \$ 6.00        |
| \$ 0.20            | \$ -           |
| 7.00%              | 7.00%          |
| 7 months           | 7 months       |

**Fixed costs**

Interest and depreciation on investment  
 Repairs (estimated at 1.5% of initial investment)  
 Insurance and taxes (estimated at 1% of investment)  
 Total fixed costs (\$ per year)  
 Fixed costs (\$ per bu.)

**Variable costs (\$ per bu.)**

Storage rental charge or service fee  
 Interest on grain inventory  
 Extra drying cost for corn  
 Extra shrinkage cost for corn  
 Extra handling and transportation cost  
 Quality deterioration  
 Electricity cost  
 Extra labor cost

**Total cost of storage**

Less selling price advantage  
**Net cost of storage (\$ per bu.)**

| On-farm Investment | Rental Storage |
|--------------------|----------------|
| \$ 703             |                |
| \$ 90              |                |
| \$ 60              |                |
| \$ 853             |                |
| \$ 0.85            |                |
|                    | \$ 0.322       |
| \$ 0.253           | \$ 0.245       |
|                    |                |
| \$ 0.065           |                |
| \$ 0.060           |                |
| \$ 1.152           |                |
| \$ 0.350           |                |
| \$ 2.73            | \$ 0.57        |
| \$ 0.200           | \$ -           |
| \$ 2.53            | \$ 0.57        |

Based on your analysis of the information in the spreadsheet, explain what recommendation you would make to the farm manager about what type of storage is best. Explain how you arrived at your recommendation.

*Sample Top-Score Response:*

**Session 1**

**Part A (use  $n = 132,000$  for number of bushels of corn)**

1. First, I need to decide which bins to buy. If the company needs 4 bins that will hold 132,000 bushels, then  $\frac{132,000}{4} = 33,000$ . The smallest bins that hold this capacity are those that hold 35,600 bushels. The dimensions for those bins are 42' by 32' (diameter by height) and will each cost \$32,525. If we include the floor (\$8100) and fan/heat (\$3225), then each bin will cost \$43,850.
2. Next, I need to find the height of the leg elevator. Its height is the sum of the bin height (32'), the cap height ( $x$ ), the vertical distance from the top of the cap to the entry point for the gravity spout ( $y$ ), and the remaining distance to the top of the leg elevator (2').

$$\text{To solve for } x: \tan 35 = \frac{x}{21} \quad \text{and to solve for } y: \tan 40 = \frac{y}{36}$$

$$x \approx 14.70 \qquad y \approx 30.21$$

Total height of the leg elevator is  $32 + 14.70 + 30.21 + 2 = 78.91$  ft.

3. Next is the length of each gravity spout ( $z$ ). Using the Pythagorean Theorem (student may choose to use right triangle trigonometry),  $36^2 + 30.21^2 = z^2$ , and solved for  $z$ . I found the length of one gravity spout to be approximately 47 ft. Since there are four of them, we will need 188 feet.
4. Finally, I now have enough information to find the total cost of the project:

**Bins** are  $4(\$43,850) = \$175,400$ .

**Leg elevator** is  $\$15,000 + \$125(78.91)$ , which is  $\$24,863.75$ .

The **gravity spouts** are  $\$20(188)$  or  $\$3,760.00$ .

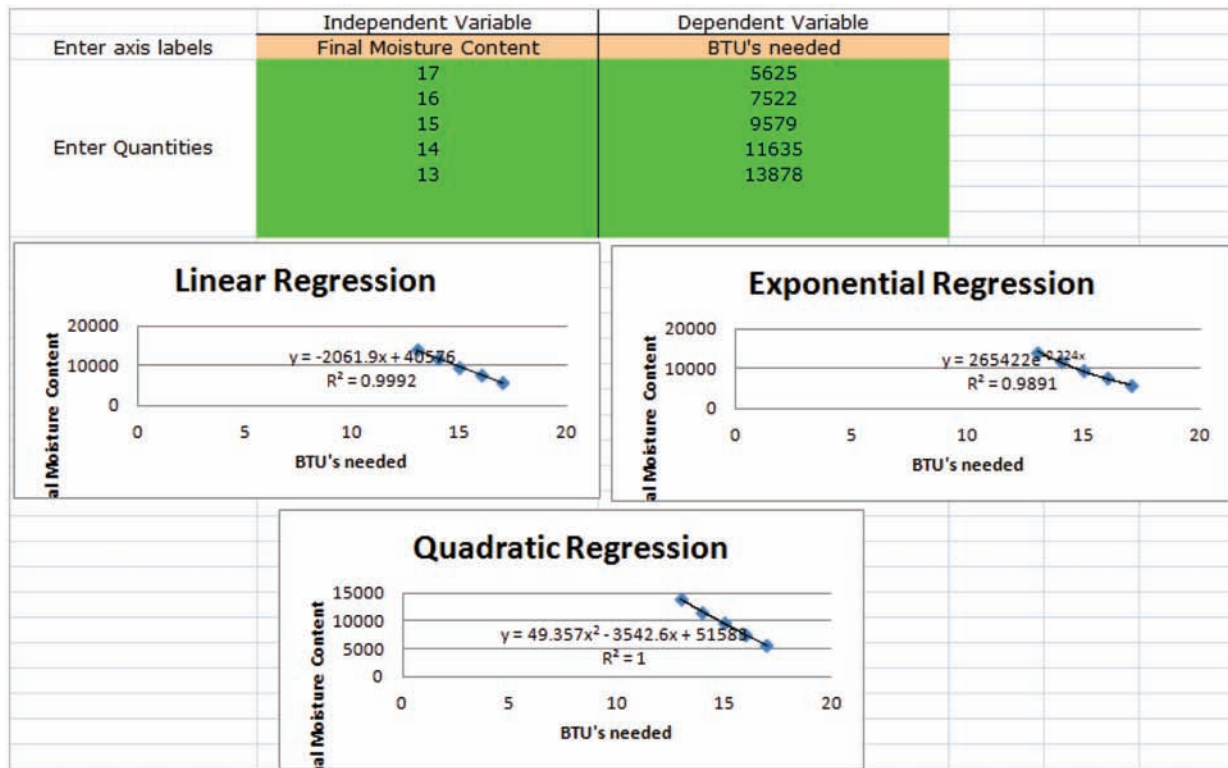
Grand total cost of the project is **\$204,023.75**.

**Part B**

Cost of drying corn (Assuming LP Gas costs \$2.18 per gallon)

HS Mathematics Sample PT Form Claim 4

Finding BTU's needed to dry corn –



Since the quadratic regression has the highest r-squared value, I will use that equation to calculate the number of BTU's needed to dry one bushel of corn from 20% to 15.5% and to 13.5%.

Quadratic regression equation:  $y = 49.357x^2 - 3542.6x + 51583$

BTU's needed for 15.5%:  $y = 49.357(15.5)^2 - 3542.6(15.5) + 51583 = 8,531$  BTU's

BTU's needed for 14% (From table): 11,635 BTU's

BTU's needed for 13.5%:  $y = 49.357(13.5)^2 - 3542.6(13.5) + 51583 = 12,753$  BTU's

Finding per bushel cost –

For 15.5%:  $8,531 \text{ BTU's} \cdot \frac{1 \text{ gallon}}{92,000 \text{ BTU's}} \cdot \frac{\$2.18}{1 \text{ gallon}} = \$0.202$  per bushel

For 14%:  $11,635 \text{ BTU's} \cdot \frac{1 \text{ gallon}}{92,000 \text{ BTU's}} \cdot \frac{\$2.18}{1 \text{ gallon}} = \$0.276$  per bushel

For 13.5%:  $12,753 \text{ BTU's} \cdot \frac{1 \text{ gallon}}{92,000 \text{ BTU's}} \cdot \frac{\$2.18}{1 \text{ gallon}} = \$0.302$  per bushel

Extra cost to dry 1 bushel of corn to 14% = cost to dry to 14% - cost to dry to 15.5%  
 =  $\$0.276 - \$0.202$   
 =  $\$0.074$

Extra cost to dry 1 bushel of corn to 13.5% = cost to dry to 13.5% - cost to dry to 15.5%  
 =  $\$0.302 - \$0.202$   
 =  $\$0.100$

**Part C**

Shrinkage cost (Assuming market price of \$6.40 per bushel)

Finding the weight of the dry material in 56 lbs of corn –

For 15.5% moisture content (84.5% dry material):

Weight of dry material =  $56(.845) = 47.32$  pounds

For 14% moisture content (86% dry material):

Weight of dry material =  $56(.86) = 48.16$  pounds

For 13.5% moisture content (86.5% dry material):

Weight of dry material =  $56(.865) = 48.44$  pounds

Finding price per pound of dry material for corn at 15.5% moisture content –

$$\frac{\$6.40}{47.32 \text{ lbs of dry material}} = \$0.135 \text{ per lb of dry material}$$

Finding the value of the dry material in 56 lbs of corn at 14% and 13.5% moisture content –

$$\text{For 14\%: } 48.16 \text{ lb of dry material} \cdot \frac{\$0.135}{1 \text{ lb of dry material}} = \$6.514$$

$$\text{For 13.5\%: } 48.44 \text{ lb of dry material} \cdot \frac{\$0.135}{1 \text{ lb of dry material}} = \$6.551$$

Finding shrinkage cost –

Shrinkage cost, per bushel, for selling corn at 14%: Value of dry material – selling price  
 $\$6.514 - \$6.40 = \$0.114$

Shrinkage cost, per bushel, for selling corn at 13.5%: Value of dry material – selling price  
 $\$6.551 - \$6.40 = \$0.151$

**Session 2**
**Part D**

The information provided in part D gives the following values that can be directly entered into the spreadsheet.

Cost to store 1 bushel of corn for 3 months in rental storage: \$0.09

Cost to store 1 bushel for each month past the initial 3 months: \$0.02

Percent of crop put in storage: September (20%), October (40%), November (30%), December (10%)

The following information can be found on the student's note sheet from the previous day's work.

Number of bushels of corn harvested: 132,000

The number of bushels that can possibly be transported must be calculated.

$$\text{For January, February, and August: } \frac{2 \text{ truckloads}}{1 \text{ day}} \times \frac{20 \text{ days}}{1 \text{ month}} \times \frac{600 \text{ bu.}}{1 \text{ truckload}} = \frac{24,000 \text{ bu.}}{\text{month}}$$

$$\text{For March, April, May, June, July: } \frac{1 \text{ truckloads}}{1 \text{ day}} \times \frac{20 \text{ days}}{1 \text{ month}} \times \frac{600 \text{ bu.}}{1 \text{ truckload}} = \frac{12,000 \text{ bu.}}{\text{month}}$$

Entering all of these values into the spreadsheet results in a total cost of \$23,160 for rental

storage to store the season's 132,000 bushels of harvested corn.

| Cost to store 1 bushel for 3 months=                             |                                | 0.09   |                                      |   |  |                        |
|--|--------------------------------|--|--------------------------------------|---|--|------------------------|
| Cost to store 1 bushel for each month past the initial 3 months= |                                | 0.02   |                                      |   |  |                        |
| Number of bushels of corn harvested =                            |                                | 132,000  |                                      |   |  |                        |
| Month  | Percent of Crop Put in Storage | Number of Bushels That It Is Possible to Transport | Percent of Crop Removed From Storage | Number of Bushels Removed From Storage During Month | Number of Bushels Remaining In Storage at End of Month | Cost of Rental Storage |
| September  | 20                             |  |                                      |   |  | \$2,376.00             |
| October  | 40                             |  |                                      |   |  | \$4,752.00             |
| November   | 30                             |  |                                      |   |  | \$3,564.00             |
| December   | 10                             |  |                                      |   |  | \$1,188.00             |
| January  |                                | 24,000   | 0.18182                              | 24,000  | 108,000  | \$2,640.00             |
| February   |                                | 24,000   | 0.18182                              | 24,000  | 84,000   | \$2,160.00             |
| March  |                                | 12,000   | 0.09091                              | 12,000  | 72,000   | \$1,680.00             |
| April  |                                | 12,000   | 0.09091                              | 12,000  | 60,000   | \$1,440.00             |
| May  |                                | 12,000   | 0.09091                              | 12,000  | 48,000   | \$1,200.00             |
| June   |                                | 12,000   | 0.09091                              | 12,000  | 36,000   | \$960.00               |
| July   |                                | 12,000   | 0.09091                              | 12,000  | 24,000   | \$720.00               |
| August   |                                | 24,000   | 0.18182                              | 24,000  | 0  | \$480.00               |
| Total Cost of Rental Storage=                                    |                                |  |                                      |   |  | \$23,160.00            |

### Part E

The students will use the current selling price for 1 bushel of shelled corn, which they will find in the pre-work session. Suppose this price was \$6.40, and the price was found when the test is taken in the month of March. Using the provided graph for the "Monthly cash prices as % of September-October price," it is possible to determine a likely value for the selling price at harvest time in September-October as follows.

The March price of corn, on average, is about 114% of the October price. This means that  $\$6.40 = 1.14(\text{October price})$ , or  $\text{October price} = \frac{\$6.40}{1.14} \approx \$5.61$ . So based on the monthly cash prices in the graph, on average, we would expect that the price at harvest would be about \$5.61. This value can be placed into the spreadsheet for the "Expected September-October selling price" on line 35.

The "Average increase in selling price" must also be calculated and put into the spreadsheet. To do this, we must use the information in the provided time series plot, and



also the values for the percent of crop removed in each month from January to August from the spreadsheet in part D.

| Month    | Percent of Crop Removed, $r$ ,<br>(from spreadsheet in part D) | Monthly Cash Price, $p$ ,<br>as % of Sept.-Oct. Price<br>(estimated from graph) | $r \times p$<br>(as a decimal) |
|----------|--|---|--------------------------------|
| January  | 18.18%   | 108%  | 0.1963                         |
| February | 18.18%   | 112%  | 0.2036                         |
| March    | 9.09%  | 114%  | 0.1036                         |
| April    | 9.09%  | 115%  | 0.1045                         |
| May      | 9.09%  | 116%  | 0.1054                         |
| June     | 9.09%  | 116%  | 0.1054                         |
| July     | 9.09%  | 113%  | 0.1027                         |
| August   | 18.18%   | 111%  | 0.2018                         |
|          |  | Total =   | 1.1233                         |

So when different percentages of corn are sold at different percentages of the Sept.-Oct. selling price, the weighted average for the corn sold is approximately 1.1233 times the Sept.-Oct. selling price, or in other words the average increase in selling price is about 12.33% higher than the harvest price. The average increase in selling price is thus approximately  $\$5.61(0.1233) = \$0.6917$ . This value can be entered for the "Average increase in selling price" on line 36 of the spreadsheet below.

Five other values must be obtained from the student's note sheet from the previous day's work. These values are the following.

Total cost of constructing the grain bins: \$203,991.25 (from part A on day 1)

Extra cost to dry 1 bu. to 13.5% moisture (for grain bin): \$0.10 (from part C on day 1)

Extra cost to dry 1 bu. to 14% moisture (for rental storage): \$0.074 (from part C on day 1)

Shrinkage Cost for selling corn at 14% moisture (for rental storage): \$0.114 (from part C on day 1)

Shrinkage Cost for selling corn at 13.5% moisture (for grain bins): \$0.151 (from part C on day 1)

These five values must be entered into the spreadsheet on the appropriate lines.

| Grain Storage Investment Comparison  |                           |                       |
|--|---------------------------|-----------------------|
| Ag Decision Maker -- Iowa State University Extension   |                           |                       |
| Place the cursor over cells with red triangles to read comments.   |                           |                       |
| Enter your input values in yellow shaded cells.  |                           |                       |
|  | Corn                      |                       |
| <b>Storage Type</b>  | <b>On-farm Investment</b> | <b>Rental Storage</b> |
| Number of bushels stored   | 132,000 bu.               | 132,000 bu.           |
| Total cost of bin construction (\$)<br>bin(including floor, fan, heat), spouts, leg elevator<br>additional augers, fans, conveyers, etc. | \$ 204,024                |                       |
| Useful life for bin (years)  | 25 years                  |                       |
| Useful life for augers, fans, conveyers (years)  | 10 years                  |                       |
| Interest rate for financing initial investment (%)   | 4.6%                      |                       |
| Average moisture at harvest (%)  | 20.0%                     | 20.0%                 |
| Moisture level for storage (%)   | 13.5%                     | 14.0%                 |
| Extra handling cost (\$ per bu.)   | \$ 0.015                  |                       |
| Extra transportation cost (\$ per bu.)   | \$ 0.050                  |                       |
| Quality loss for on-farm storage (%)   | 1.0%                      |                       |
| Fans, total horsepower   | 10 hp                     |                       |
| Hours fans will run, total   | 1440 hrs                  |                       |
| Electricity rate (\$ per KWH)  | \$ 0.1000                 |                       |
| Add'l hours per month spent managing & handling grain  | 5.00                      |                       |
| Labor value (\$ per hour)  | \$ 10.00                  |                       |
| Commercial storage charges   |                           |                       |
| Minimum charge or base rate (\$ per bu.)   |                           | \$0.0900              |
| Base period (months)   |                           | 3 months              |
| Rate per month after base period (\$ per bu. per month)  |                           | \$0.0200              |
| Expected September-October cash selling price of corn (\$ per bu.)   | \$ 5.61                   | \$ 5.61               |
| Average selling price advantage (\$ per bu.)   | \$ 0.69                   | \$ 0.49               |
| Short-term interest rate (%)   | 7.00%                     | 7.00%                 |
| Average length of storage (months)   | 7 months                  | 7 months              |
|  | <b>On-farm Investment</b> | <b>Rental Storage</b> |
| <b>Fixed costs</b>   |                           |                       |
| Interest and depreciation on investment  | \$ 14,534                 |                       |
| Repairs (estimated at 1.5% of initial investment)  | \$ 3,135                  |                       |
| Insurance and taxes (estimated at 1% of investment)  | \$ 2,090                  |                       |
| Total fixed costs (\$ per year)  | \$ 19,759                 |                       |
| Fixed costs (\$ per bu.)   | \$ 0.15                   |                       |
| <b>Variable costs (\$ per bu.)</b>   |                           |                       |
| Storage rental charge or service fee   |                           | \$ 0.170              |
| Interest on grain inventory  | \$ 0.257                  | \$ 0.249              |
| Extra drying cost for corn   | \$ 0.100                  | \$ 0.074              |
| Extra shrinkage cost for corn  | \$ 0.151                  | \$ 0.114              |
| Extra handling and transportation cost   | \$ 0.065                  |                       |
| Quality deterioration  | \$ 0.056                  |                       |
| Electricity cost   | \$ 0.009                  |                       |
| Extra labor cost   | \$ 0.003                  |                       |
| <b>Total cost of storage</b>   | <b>\$ 0.79</b>            | <b>\$ 0.61</b>        |
| Less selling price advantage   | \$ 0.692                  | \$ 0.492              |
| <b>Net cost of storage (\$ per bu.)</b>  | <b>\$ 0.10</b>            | <b>\$ 0.12</b>        |

## HS Mathematics Sample PT Form Claim 4

Recommendation: Based strictly on cost, the best recommendation to make to the farmer would be to build the grain bins and use them for storage, since the net cost for rental storage is \$0.12 per bushel and the net cost for grain bin storage is \$0.10 per bushel, which is lower. However, other considerations might convince the farmer to accept the higher cost.

*Scoring Notes:*

*Scoring Rubric:*

Responses to Part A will receive 0-4 points based on the following:

**4 points:** The student demonstrates a thorough understanding of the 3 major concepts assessed in this part: complete investigation of choice in size of the bins; use of right triangle trigonometry (or Pythagorean Theorem) to calculate missing heights; and a combination of strategies used in earlier grades (linear functions and proportional relationships) to analyze costs.

**3 points:** The student demonstrates a thorough understanding of 2 of the 3 major concepts assessed in this part and a limited understanding of the 3<sup>rd</sup>. This limited understanding could be an inappropriate choice for the size of the bins by picking one that doesn't hold enough, OR the student will demonstrate a thorough understanding of all 3 of the major concepts assessed in this part with minor arithmetic errors.

**2 points:** The student demonstrates a thorough understanding of 1 of the 3 major concepts assessed in this part and a limited understanding of the other two. A student receiving 2 points for this part may thoroughly determine the correct size of the bins but makes significant errors in the other two parts.

**1 point:** The student demonstrates a limited understanding of all of the 3 major concepts assessed in this part OR a thorough understanding of 1 of the 3 concepts and little to no understanding of the other 2. A student receiving 1 point for this part may thoroughly determine the correct size of the bins but only be able to guess at a cost for the rest of the project based on conjecture.

**0 points:** The student demonstrates little to no understanding of any of the 3 major concepts assessed in this part.

Responses to Part B will receive 0-4 points based on the following:

**4 points:** The student has a thorough understanding of how to analyze a real-world scenario to identify important quantities and use units to solve problems. The student has a thorough understanding of how to select and use a regression model in the context of the data. The student enters the values for final moisture content as the independent variable and the values for the number of BTU's needed to dry from 20% moisture content as the dependent variable. The student identifies that the quadratic regression is the best fit or explains that another type of regression is close enough to a perfect fit that the level of error would be negligible. The student uses the chosen regression function to find the number of BTU's needed to dry the corn to 15.5% and to 13.5% moisture content. The student shows how the units of the quantities lead to the calculation for the total cost per

## HS Mathematics Sample PT Form Claim 4

bushel of drying corn and then subtracts to find the extra cost for drying corn to 14% and to 13.5%.

**3 points:** The student has an adequate understanding of how to analyze a real-world scenario to identify important quantities and use units to solve problems. The student has a thorough understanding of how to select and use a regression model in the context of the data. The student enters the values for final moisture content as the independent variable and the values for the number of BTU's needed to dry from 20% moisture content as the dependent variable. The student identifies that the quadratic regression is the best fit or explains that another type of regression is close enough to a perfect fit that the level of error would be negligible. The student uses the chosen regression function to find the number of BTU's needed to dry the corn to 15.5% and to 13.5% moisture content. The student shows how the units of the quantities lead to the calculation for the total cost per bushel of drying corn but forgets to subtract to find the extra cost for drying corn to 14% and to 13.5%.

**2 points:** The student has a solid understanding of how to analyze a real-world scenario to identify important quantities and use units to solve problems. The student has a limited understanding of how to select and use a regression model in the context of the data. The student either does not correctly use the regression spreadsheet to identify the best model for the data, or uses a model other than the quadratic one without explaining why it is acceptable in the context. The student uses the values they identified (which may be incorrect) for the number of BTU's needed to dry the corn to each level and shows how the units of the quantities lead to the calculation for the total cost per bushel of drying corn and then subtracts to find the extra cost for drying corn to 14% and to 13.5%.

**1 point:** The student has some understanding of how to analyze a real-world scenario to identify important quantities and use units to solve problems. The student has a limited understanding of how to select and use a regression model in the context of the data. The student either does not correctly use the regression spreadsheet to identify the best model for the data, or uses a model other than the quadratic one without explaining why it is acceptable in the context. The student uses the values they identified (which may be incorrect) for the number of BTU's needed to dry the corn to each level and shows how the units of the quantities lead to the calculation for the total cost per bushel of drying corn but forgets to subtract to find the extra cost for drying corn to 14% and to 13.5%. **OR** The student has limited understanding of how to analyze a real-world scenario to identify important quantities and use units to solve problems. The student has a solid understanding of how to select and use a regression model in the context of the data. The student enters the values for final moisture content as the independent variable and the values for the number of BTU's needed to dry from 20% moisture content as the dependent variable. The student identifies that the quadratic regression is the best fit or explains that another type of regression is close enough to a perfect fit that the level of error would be negligible. The student does not use the values they identified (which are correct) for the number of BTU's needed to dry the corn to each level to show how the units of the quantities lead to the calculation for the total cost per bushel of drying.

**0 points:** The student has limited understanding of how to analyze a real-world scenario to identify important quantities and use units to solve problems. The student has a limited understanding of how to select and use a regression model in the context of the data. The student either does not correctly use the regression spreadsheet to identify the best model for the data, or uses a model other than the quadratic one without explaining why it is acceptable in the context. The student either does not identify the number of BTU's needed

to dry the corn to each level or does not use the values they identified (which are incorrect) to show how the units of the quantities lead to the calculation for the total cost.

Responses to Part C will receive 0-4 points based on the following:

**4 points:** The student has a thorough understanding of how to analyze a real-world scenario to calculate rates and use units to solve problems. The student uses the percent of moisture content to calculate the weight of the dry material in 56 lbs of corn at each moisture level. The student uses the current price of corn to calculate the value of the dry material in the corn at the market standard moisture level of 15.5%. The student uses the calculated rate to find the value of the dry material in 56 lbs of each of the dryer corns. The student compares the value of the dry material in each of the dryer corns to the selling price to find the cost of shrinkage.

**3 points:** The student has some understanding of how to analyze a real-world scenario to calculate rates and use units to solve problems. The student uses the percent of moisture content to calculate the weight of the dry material in 56 lbs of corn at each moisture level. The student uses the current price of corn to calculate the value of the dry material in the corn at the market standard moisture level of 15.5%. The student uses the calculated rate to find the value of the dry material in 56 lbs of each of the dryer corns. The student does not compare the value of the dry material in each of the dryer corns to the selling price to find the cost of shrinkage.

**2 points:** The student has incomplete understanding of how to analyze a real-world scenario to calculate rates and use units to solve problems. The student uses the percent of moisture content to calculate the weight of the dry material in 56 lbs of corn at each moisture level. The student uses the current price of corn to calculate the value of the dry material in the corn at the market standard moisture level of 15.5%. The student does not use the calculated rate to find the value of the dry material in 56 lbs of each of the dryer corns, and so cannot compare the value of the dry material in each of the dryer corns to the selling price to find the cost of shrinkage.

**1 point:** The student has limited understanding of how to analyze a real-world scenario to calculate rates and use units to solve problems. The student uses the percent of moisture content to calculate the weight of the dry material in 56 lbs of corn at each moisture level. The student does not use the current price of corn to calculate the value of the dry material in the corn at the market standard moisture level of 15.5%. The student cannot find the value of the dry material in 56 lbs of each of the dryer corns, and so cannot compare the value of the dry material in each of the dryer corns to the selling price to find the cost of shrinkage.

**0 points:** The student has no understanding of how to analyze a real-world scenario to calculate rates and use units to solve problems. The student does not use the percent of moisture content to calculate the weight of the dry material in 56 lbs of corn at each moisture level. The student therefore cannot find the value of the dry material and the cost of shrinkage.

Responses to Part D will receive 0-4 points based on the following:

**4 points:** The student has a thorough understanding of how to analyze a real-world scenario to calculate rates and use units to solve problems. The student correctly calculates

the number of bushels of corn that can be transported to market for the block of months January, February, August, and separately for the block of months March, April, May, June, July. The student identifies the appropriate information to enter into the provided spreadsheet, enters this information into the spreadsheet, and records the value of \$23,160 for the total cost of transportation.

**3 points:** The student has some understanding of how to analyze a real-world scenario to calculate rates and use units to solve problems. The student fails to correctly calculate the amount of corn transported for one of the block of months, but correctly calculates it for the other block of months. The student identifies the appropriate information to enter into the provided spreadsheet, enters this information into the spreadsheet, and records a value for the total cost of transportation that is correct except for the use of the one incorrect number.

**2 points:** The student has incomplete understanding of how to analyze a real-world scenario to calculate rates and use units to solve problems. The student fails to correctly calculate the amount of corn transported for both of the blocks of months. The student identifies the appropriate information to enter into the provided spreadsheet, enters this information into the spreadsheet, and records a value for the total cost of transportation that is correct except for the use of the two incorrect numbers.

**1 point:** The student has limited understanding of how to analyze a real-world scenario to calculate rates and use units to solve problems. The student fails to correctly calculate the amount of corn transported for both of the blocks of months. The student enters some information correctly into the spreadsheet, but not all correct information, and thus records an incorrect number for the total cost of transportation.

**0 points:** The student has no understanding of how to analyze a real-world scenario to calculate rates and use units to solve problems. The student fails to perform any calculation correctly, and fails to enter any correct information into the spreadsheet.

Responses to Part E will receive 0-4 points based on the following:

**4 points:** The student has a thorough understanding of how to analyze a real-world scenario to read information on a graph, set up a simple linear equation and solve for an unknown value, and reason quantitatively using percents. The student uses the current selling price of corn and the provided time series plot to correctly calculate the estimated selling price of corn at harvest time in October. The student also correctly calculates the "Average selling price advantage" by using the percent of crop removed and sold each month and the monthly cash price during that month to multiply percents and then calculate an average percent above the October price. These values and values from the previous day are all entered correctly into the spreadsheet, and then the student makes a recommendation about what type of storage to use and gives valid reasons for the recommendation.

**3 points:** The student has some understanding of how to analyze a real-world scenario to read information on a graph, set up a simple linear equation and solve for an unknown value, and reason quantitatively using percents. The student correctly calculates the "Average selling price advantage" by using the percent of crop removed and sold each month and the monthly cash price during that month to multiply percents and then calculate an average percent above the October price. However, the student fails to use the current selling price of corn and the provided time series plot to correctly calculate the

estimated selling price of corn at harvest time in October. These values and values from the previous day are all entered correctly into the spreadsheet, and then the student makes a recommendation about what type of storage to use and gives valid reasons for the recommendation.

**2 points:** The student has some understanding of how to analyze a real-world scenario to read information on a graph, set up a simple linear equation and solve for an unknown value, and reason quantitatively using percents. The student uses the current selling price of corn and the provided time series plot to correctly calculate the estimated selling price of corn at harvest time in October. However, the student fails to correctly calculate the "Average selling price advantage" by using the percent of crop removed and sold each month and the monthly cash price during that month to multiply percents and then calculate an average percent above the October price. These values and values from the previous day are all entered correctly into the spreadsheet, and then the student makes a recommendation about what type of storage to use and gives valid reasons for the recommendation.

**1 points:** The student has limited understanding of how to analyze a real-world scenario to read information on a graph, set up a simple linear equation and solve for an unknown value, and reason quantitatively using percents. The student fails to use the current selling price of corn and the provided time series plot to correctly calculate the estimated selling price of corn at harvest time in October. The student also fails to correctly calculate the "Average selling price advantage" by using the percent of crop removed and sold each month and the monthly cash price during that month to multiply percents and then calculate an average percent above the October price. The student makes a recommendation about what storage to use, but the recommendation is made based on incorrectly calculated numbers from the spreadsheet.

**0 points:** The student has no understanding of how to analyze a real-world scenario to read information on a graph, set up a simple linear equation and solve for an unknown value, and reason quantitatively using percents. Any calculations made are incorrect, and no recommendation is made or a recommendation is made but no reasoning is given to justify it.

**MAT.HS.PT.4.CANSB.A.051**

|                              |  |
|------------------------------|--|
| Sample Item ID:              | MAT.HS.PT.4.CANSB.A.051  |
| Title:                       | Packaging Cans   |
| Grade:                       | HS   |
| Primary Claim:               | <b>Claim 4: Modeling and Data Analysis</b><br>Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.  |
| Secondary Claim(s):          | <p>Claim 2: Problem Solving<br/>Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.</p> <p>Claim 1: Concepts and Procedures<br/>Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.</p>   |
| Primary Content Domain:      | <b>Geometry</b>  |
| Secondary Content Domain(s): | Algebra, Functions, Statistics, and Probability  |
| Assessment Target(s):        | <p>4A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.</p> <p>4E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.</p> <p>4D: Interpret results in the context of a situation.</p> <p>4G: Identify, analyze, and synthesize relevant external resources to pose or solve problems.</p> <p>4B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.</p> <p>2B: Select and use appropriate tools strategically.</p> <p>1G: Create equations that describe numbers or relationships.</p> <p>1H: Understand solving equations as a process of reasoning and explain the reasoning.</p> <p>1L: Interpret functions that arise in applications in terms of a context.</p> <p>1P: Summarize, represent, and interpret data on a single count or measurement variable.</p> |



|   |  |
|---|--|
|   | <p>1H (Gr 8): Understand and apply the Pythagorean theorem.</p> <p>1I (Gr 8): Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.</p>  |
| Standard(s):  | A-CED.2, A-CED.4, A-REI.2, F-IF.4, G-GMD.3, G-MG.3, S-ID.1, S-IC.1, 8.G.7, 8.G.9   |
| Mathematical Practice(s):   | 1, 2, 3, 4, 5, 6   |
| DOK:  | 4  |
| Item Type:  | PT   |
| Score Points:   | 20   |
| Difficulty:   | H  |
| How this task addresses the “sufficient evidence” for this claim: | The student uses concepts of geometry, functions, and statistical analysis to determine appropriate arrangements and measures that will minimize waste and cost. Additionally, the student must provide mathematical justifications to support reasoning.  |
| Target-specific attributes (e.g., accessibility issues):          | Accommodations may be necessary for students who have visual challenges.   |
| Stimulus/Source:  | <a href="http://zunal.com/webquest.php?w=4309">http://zunal.com/webquest.php?w=4309</a><br><a href="http://math.arizona.edu/~vpiercey/PackingEfficiency.pdf">http://math.arizona.edu/~vpiercey/PackingEfficiency.pdf</a><br><a href="http://www.cancentral.com/howmade.cfm#twopiece">http://www.cancentral.com/howmade.cfm#twopiece</a><br><a href="http://answers.google.com/answers/threadview/id/601197.html">http://answers.google.com/answers/threadview/id/601197.html</a>   |
| Notes:  | Multiple sessions  |
| Task Overview:  | The student assumes the role of consultant to the president of a beverage company. In class and individually, the student completes tasks in which he/she investigates the impact on the amount of space used in a box with different arrangements of the cans in the box. This investigation is done in class using spreadsheets specifically designed to compute measures. Students also investigate this analytically in their individual work. The student further explores minimizing cost to the company by determining a function for this purpose based on given information. Finally, the student provides statistical reasoning to make a valid argument based on data provided. |
| Teacher preparation / Resource requirements:                      | <p>Teacher preparation:</p> <p>Up to two school days prior to administration of the task, students must be assigned a prework task that will be used to help their understanding of the objectives of the task itself. Students must have pre-work ready to be shared at the start of the task. Session 1 of the task will start with students being divided into groups of 3 or 4 to complete Part A. Afterwards, results of the group work will be discussed as a class. The remainder of session 1 will include Part B, and should be completed individually. During session 2, Parts C and D should be completed individually.</p>   |

|  |   |
|--|---|
|  | <p>Resource requirements:<br/>                 Students will need to access graphing calculator software and statistical software provided in the tasks. Or they will need to be provided with other tools in which they can organize data into a box plot. The tool(s) the students use should allow for finding the minimum value of a function within a set region. Furthermore, spreadsheet software should be available to students in Part C of the task. This part allows students to receive hint(s) if they have difficulty approaching and solving the given problem. Should the student use this option, he/she will receive fewer points for their answer, depending on the number of hints they choose to use.</p> |
| <p>Teacher Responsibilities During Administration:</p> | <p>Monitor individual student work; provide resources as necessary.</p>   |
| <p>Time Requirements:</p>                              | <p>One prework assignment is given up to two school days prior to starting this task. The prework will not be scored. Two sessions of the task, including both group work and individual work, will total no more than 120 minutes. All portions of the task will be scored, with the exception of the group work in Part A.</p>  |

## Packaging Cans

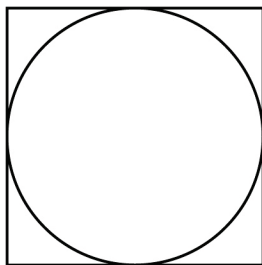
### ***Prework***

[Up to two school days prior to starting the performance task, teachers should assign the following work to students. This prework must be brought to class on the day the performance task begins.]

Perform a search to find the dimensions, in centimeters, of a standard-sized soda (pop) can. Identify the radius of the circular base of the can and the height of the can.

radius = \_\_\_\_\_ cm      height = \_\_\_\_\_ cm

Imagine a circle fit inside a square so that it touches each side of the square, as shown in this diagram.



The circle has the same radius as the soda can whose dimensions you identified above. Find the area, in square centimeters, inside the square that represents the area outside the circle.

$$\text{Area} = \underline{\hspace{2cm}} \text{ cm}^2$$

Write the formula for the surface area and volume of a right circular cylinder.

$$\text{Surface Area} = \underline{\hspace{4cm}}$$

$$\text{Volume} = \underline{\hspace{4cm}}$$

## Packaging Cans

### Session 1

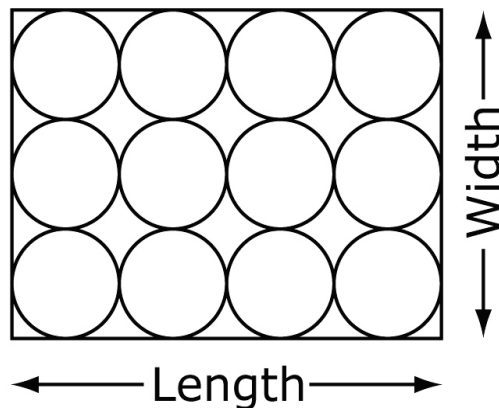
#### **Part A** (Group work)

[Session 1 of the task will start with group work. Students will be divided into groups of 3 or 4 and work for about 20 minutes using part of their pre-work assignment to explore the relationships among different ways to stack cans in a box. This group work will not be scored.]

You have been asked to be a consultant for a beverage company. The company president would like you to investigate how soda cans are packaged. Cans are constructed in such a way that they are not truly cylinders, but for the purpose of your investigation, we will assume that they are right circular cylinders.

The current boxes used to package soda cans have rectangular bases. The 12 cans in a box are stacked in one layer. The diagram below shows Stacking Method A, a 3-can by 4-can arrangement.

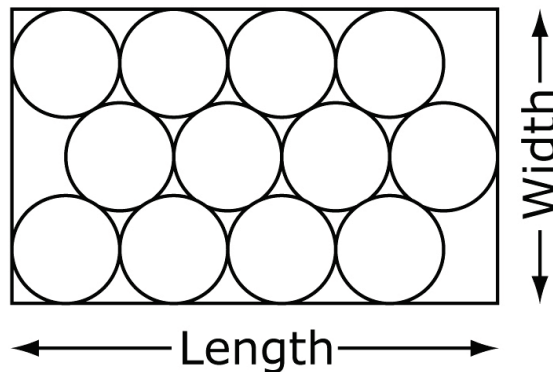
### Stacking Method A



With your group, find all possible one-layer stacking arrangements for 12 cans in a rectangular box where the cans touch as shown. Show them in the space below. The number of cans along the length and the width must be factors of 12.

The president of the beverage company shows a preference for a 3-can by 4-can arrangement. The president suggests an alternate way of stacking the 12 cans in a box, using Stacking Method B, shown below.

### Stacking Method B



Use the spreadsheet below to compare the different stacking methods of 12 cans. In the top portion of the spreadsheet, enter appropriate values into the highlighted spaces. The spreadsheet will calculate the parts in the bottom portion based on the values you entered.

[Table presented to students]

| <b>Stacking Cans</b>                                |                 |  |  |                 |
|---|-----------------|--|--|-----------------|
|   | <b>Method A</b> |  |  | <b>Method B</b> |
| Radius of can (cm)                                  |                 |  |  |                 |
| Number of rows                                      | 3               |  |  |                 |
| Cans in each row                                    | 4               |  |  |                 |
|   |                 |  |  |                 |
| Base area of one can                                |                 |  |  |                 |
| Base area of 12 cans                                |                 |  |  |                 |
| Box length (cm)                                     |                 |  |  |                 |
| Box width (cm)                                      |                 |  |  |                 |
| Base area of box (cm <sup>2</sup> )                 |                 |  |  |                 |
| Area of base NOT covered by cans (cm <sup>2</sup> ) |                 |  |  |                 |
| Percent of base area NOT used by cans               |                 |  |  |                 |

In your groups answer the following:

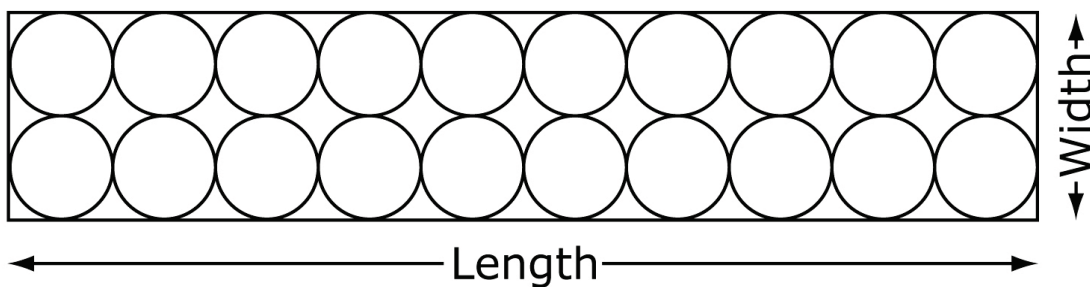
- What are some similarities in the quantities you see in the spreadsheet?
- What are some differences in the quantities?
- What do you think these quantities suggest about the efficiency of the different stacking methods?

### Cans in a Box

#### Part B

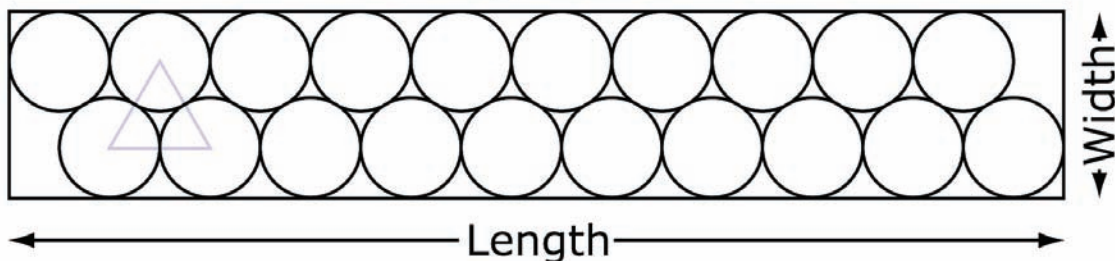
The beverage company is planning to put 20 cans in a box, stacked in one layer. They have asked you to do an analysis to determine the best way to arrange the cans to minimize wasted space and packaging materials. The diagram below represents one arrangement proposed by the company.

#### Arrangement X



A second arrangement of the 20 cans, shown below, uses a different amount of space in the base of the box.

#### Arrangement Y



Note: The triangle outlined inside Arrangement Y might help with finding the dimensions of that arrangement.

Answer the following questions about these two arrangements and provide justifications for those answers. For each question, you may use a combination of diagrams, mathematical expressions/equations, and words to justify your response.

1. Which of these two arrangements, X or Y, has less wasted space?

Click on one:    X        Y

[By clicking on either X or Y, the response will be highlighted.]

Justification:

2. Does it follow that a box whose arrangement has less wasted space also has a smaller surface area?

Click on one:        YES            NO

[By clicking on either yes or no, the response will be highlighted.]

Justification:



3. Show a third way to arrange the 20 cans that would result in a box that has a smaller surface area than the box proposed by the company, as shown by Arrangement X.

Justification:

## Session 2

### Size of the Can

#### ***Part C***

The president of the beverage company wants to minimize the cost involved in the production of standard cans.

1. Calculate the surface area, in square centimeters, and volume, in cubic centimeters, of a standard can that has a radius of 3.3 cm and a height of 12 cm.

Surface Area = \_\_\_\_\_

Volume = \_\_\_\_\_

2. Find the radius and height, in centimeters, of the cylindrical can that would contain the same volume as a standard can but would minimize the surface area. You may use the graphing calculator from the link below in your investigation.

<http://www.shodor.org/interactivate/activities/FunctionFlyer/>

radius = \_\_\_\_\_ cm      height = \_\_\_\_\_ cm

**Variation:** For successively greater deductions in total points possible, student may ask for up to 3 "hints" and receive them in the order shown below.

Hint 1: The formula for the surface area of a cylinder is  $S = 2\pi r^2 + 2\pi rh$ . The formula for the volume of a cylinder is  $V = \pi r^2 h$ . Use the volume you calculated for a standard can and the volume formula to solve for the height,  $h$ . Then use the resulting expression for  $h$  in the surface area formula to determine a function that can be used to find the radius for the can with the minimum surface area.

Hint 2: The function  $y = 2\pi x^2 + \frac{821.08}{x}$  represents the surface area of a cylinder in terms of its radius. Graph this function and find the minimum value of  $y$ .

Hint 3: An alternative strategy is to estimate the solution by substituting possible values for the radius. You can use the "guess and check" table below. Enter different values for the radius in the highlighted spaces. The table will calculate the values for height and surface area of the can.

| Radius (cm) | Height (cm) | Surface Area (cm <sup>2</sup> ) |
|-------------|-------------|---------------------------------|
|             |             |                                 |
|             |             |                                 |
|             |             |                                 |
|             |             |                                 |
|             |             |                                 |
|             |             |                                 |
|             |             |                                 |
|             |             |                                 |
|             |             |                                 |
|             |             |                                 |

3. Would you recommend to the president of the beverage company changing the dimensions of the can based on your results above? How would you convince the president that your recommendation is valid? Justify your answer in the space below.

## Is this Unusual?

### **Part D**

You suspect that one of your competitors, "Big-Jump Soda," is under-filling their cans of soda. You decide that you will purchase a random sample of 30 cans of "Big-Jump," measure the contents, and draw a conclusion based on your results.

Describe a method for collecting the 30 cans to be used in your random sample.

The following is the amount of soda, in milliliters, of the 30 cans from your sample.

355, 354, 354, 354, 354, 352, 355, 351, 357, 351,  
355, 355, 355, 356, 354, 353, 353, 352, 354, 355,  
352, 354, 355, 354, 354, 355, 355, 352, 352, 355

Organize your data into a box plot. You may use either of the online tools shown below.

<http://www.alcula.com/calculators/statistics/box-plot/>

<http://www.shodor.org/interactivate/activities/BoxPlot/>

The advertised amount of soda in a regular can is 355 ml. Based on the results of your study, do you think that “Big-Jump” is under-filling their cans? Be sure to use statistics and your graph to support your conclusion.

**End of Session 2**

*Sample Top-Score Response:*

**Part A** (Group work; not scored)

Arrangements should be shown for 3 rows of 4 (or 4 rows of 3), 2 rows of 6 (or 6 rows of 2), and 1 row of 12 (or 12 rows of 1).

[Completed table]

| <b>Stacking Cans</b>                                |                 |        |        |                 |
|---|-----------------|--------|--------|-----------------|
|   | <b>Method A</b> |        |        | <b>Method B</b> |
| Radius of can (cm)                                  | 3.3             | 3.3    | 3.3    | 3.3             |
| Number of rows                                      | 3               | 2      | 1      | 3               |
| Cans in each row                                    | 4               | 6      | 12     | 4               |
| Base area of one can                                | 34.21           | 34.21  | 34.21  | 34.21           |
| Base area of 12 cans                                | 410.54          | 410.54 | 410.54 | 410.54          |
| Box length (cm)                                     | 26.4            | 39.6   | 79.2   | 29.70           |
| Box width (cm)                                      | 19.8            | 13.2   | 6.6    | 18.03           |
| Base area of box (cm <sup>2</sup> )                 | 522.72          | 522.72 | 522.72 | 535.54          |
| Area of base NOT covered by cans (cm <sup>2</sup> ) | 112.18          | 112.18 | 112.18 | 124.99          |
| Percent of base area NOT used by cans               | 21.46%          | 21.46% | 21.46% | 23.34%          |

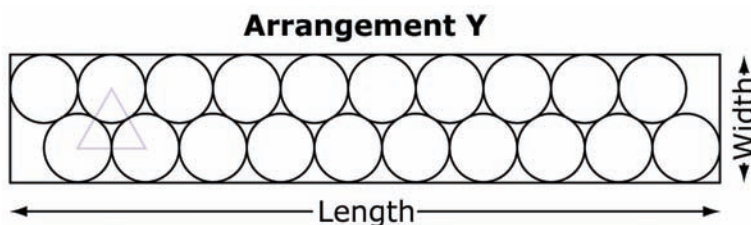
All variations of both methods have the same soda can radius, and as a result, the same base area of one and 12 cans. All variations in method A have the same base area of the box, and as a result, the same area and percent of the base not covered by cans.

Differences exist between methods A and B in the dimensions of the box, and as a result, the areas of the base of each box. Since the areas of the bases of both boxes are different, the area and percent of the bases' areas not covered by cans are also different.

The different values for both methods A and B suggest that different arrangements of cans cause box sizes to be different. And different-sized boxes, each containing the same number of cans, are going to have different amounts of wasted space.

**Part B**

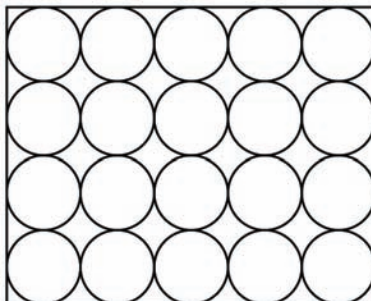
1. For arrangement X, the base of the box has length  $20r$  and width  $4r$ , where  $r$  is the length of the radius of a can. Therefore, the area of the base of the box is  $80r^2$ . Since the total area of the base of the cans in this arrangement is  $20\pi r^2$ , then the percent of the base of the box covered by the cans in this arrangement is  $\frac{20\pi r^2}{80r^2} \approx .7854$  or 78.54%.



For arrangement Y, the length is  $21r$ . The height of the triangle can be found by using the Pythagorean theorem in which the length of the hypotenuse is  $2r$  and the length of the other leg is  $r$ . Therefore, the height of the triangle is  $\sqrt{(2r)^2 - r^2}$  or  $\sqrt{3}r$ . Thus, the width of this arrangement is  $2r + \sqrt{3}r$ . The area of the base of the box for arrangement Y is then  $42r^2 + 21\sqrt{3}r^2$ . The base of the cans in this arrangement has area  $20\pi r^2$ . As a result, the percent of the base of the box that is covered by the cans in this arrangement is  $\frac{20\pi r^2}{42r^2 + 21\sqrt{3}r^2} \approx 80.17\%$ . Since arrangement Y uses more space in the base of the box than arrangement X, arrangement Y has less wasted space than arrangement X, proposed by the company.

2. The dimensions of the box with arrangement X are  $20r$  by  $4r$  by 12. Since  $r = 3.3$ , the dimensions are 66 by 13.2 by 12. So, the surface area of the box with arrangement X is  $2(66 \times 13.2 + 66 \times 12 + 13.2 \times 12) = 3643.2 \text{ cm}^2$ . The dimensions of the box with arrangement Y are  $21r$  by  $2r + \sqrt{3}r$  by 12. Since  $r = 3.3$ , the dimensions are 69.3 by 12.3 by 12. So, the surface area of the box with arrangement Y is  $2(69.3 \times 12.3 + 69.3 \times 12 + 12.3 \times 12) = 3663.18 \text{ cm}^2$ . Arrangement X has a smaller surface area but more wasted space. Therefore, it does not follow that a box with less wasted space will have a smaller surface area.

3. One example, shown here, includes a 5-can by 4-can arrangement with dimensions 33 by 26.4 by 12. The surface area of this box would be  $2(33 \times 26.4 + 33 \times 12 + 26.4 \times 12) = 3,168 \text{ cm}^2$ .

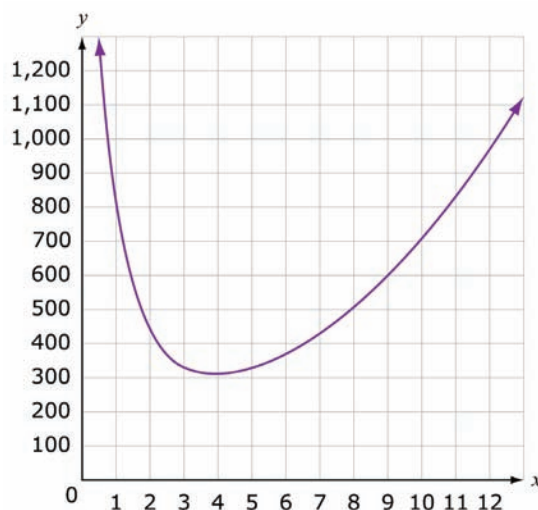


**Part C**

1. For a standard can with the given dimensions, the surface area is approximately  $2\pi(3.3)(12) + 2\pi r^2 = 317.24 \text{ cm}^2$ . The volume is approximately  $\pi(3.3)^2(12) = 410.54 \text{ cm}^3$ .
2. Since the desired volume of the can is  $410.54 \text{ cm}^3$ , the formula for the surface area of the can may be used to determine the function in the following manner:

$$\left. \begin{array}{l} V = 410.54 = \pi r^2 h \Rightarrow h = \frac{410.54}{\pi r^2} \\ SA = 2\pi r^2 + 2\pi rh \end{array} \right\} \Rightarrow SA = 2\pi r^2 + 2\pi r \frac{410.54}{\pi r^2} = 2\pi r^2 + \frac{821.08}{r}$$

Using a graphing tool, the function can be graphed for the positive values of  $x$ , since the radius of the can must be positive. The graphing tool can be used to find the minimum of the graph for positive  $x$ -values. The minimum occurs where the radius is approximately 4.03 centimeters and the height would then be equivalent to  $\frac{410.54}{\pi(4.03^2)} \approx 8.05$  centimeters.





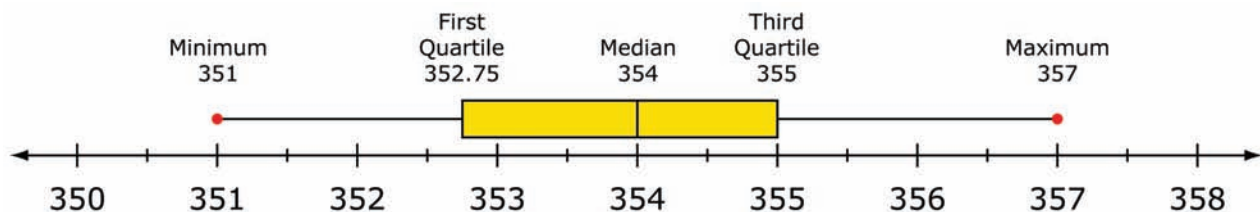
[Sample table from hint #3]

| Radius (cm) | Height (cm) | Surface Area (cm <sup>2</sup> ) |
|-------------|-------------|---------------------------------|
| 2           | 32.67       | 435.67                          |
| 2.5         | 20.91       | 367.70                          |
| 3           | 14.52       | 330.24                          |
| 3.5         | 10.67       | 311.56                          |
| 4           | 8.17        | 305.80                          |
| 4.2         | 7.41        | 306.33                          |
| 4.4         | 6.75        | 308.25                          |
| 4.6         | 6.18        | 311.45                          |
| 4.8         | 5.67        | 315.82                          |
| 5           | 5.23        | 321.30                          |

3. There are many valid responses, and points will be awarded based on the meaningful justification provided. The response should include mathematically supported reasons that benefit the company and not student preference. The surface areas of the standard-sized can and the can with minimum surface area should be compared. The student should take into account that the can with the smaller surface area has new dimensions. This will impact the amount of wasted space inside packaged boxes, and will also affect the surface area of the box.

**Part D**

1. There are many different possible responses. The method described must take into account that the sample should be random. For example, the 30 cans should not all come from the same store or even from the same region, town, or state.



2. Based on the data and box plot for the data, it appears that the company is under-filling their soda cans. The graph shows a median of 354 ml. This indicates that more than half of the cans have less than the advertised amount of soda inside.

**Scoring Notes:**

Each part of the task is evaluated individually. The total number of points is determined by adding the points assigned for each part (except *Part A*).

**Scoring Rubric:**

**Part A** Not scored. While it is desirable that the student followed the directions to create a picture that uses at least one of each colored shape with no overlap, the resulting picture has no measureable value in terms of fractional sense and understanding areas with respect to shapes.

Scoring Rubric for **Part B** Question 1: Responses to this item will receive 0-3 points, based on the following:

**3 points:** The student shows a thorough understanding of how to find the dimensions of each box in arrangements X and Y and the amount of wasted space in both boxes. The student knows to and correctly applies the Pythagorean theorem to find the height of the triangle shown on arrangement Y to determine the dimensions of that box. The student fully understands the process for comparing the space used in both arrangements by correctly calculating and comparing the spaces used: 78.54% and 80.17%. The student correctly identifies arrangement Y as having less wasted space.

**2 points:** The student shows some understanding of how to find the dimensions of each box in arrangements X and Y and the amount of wasted space in both boxes. The student applies the Pythagorean theorem to help determine the dimensions of the box in arrangement Y, but makes an error in calculating one of the dimensions of either box or the area of either box. The student understands the process for calculating and comparing the amount of wasted space in the boxes for both arrangements.

**1 point:** The student shows partial understanding of how to find the dimensions of each box in arrangements X and Y and the amount of wasted space in both boxes. The student either does not apply the Pythagorean theorem to determine a dimension of the box in arrangement Y, or the student makes more than one error in calculating the dimensions or areas of either arrangement's bases. The student understands the process for comparing the amount of wasted space in the boxes for both arrangements. **OR** The student understands the process for calculating the amount of wasted space inside each box but makes some errors calculating the dimensions of each box, the areas of each base, and/or amount of space used inside the boxes.

**0 points:** The student shows inconsistent understanding of how to find the dimensions of each box in arrangements X and Y and the amount of wasted space in both boxes. The student does not correctly find the dimensions of either box and the student does not correctly calculate the amount of wasted space in each box.

**Part B**, Question 2: Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student shows a thorough understanding of how to apply the concepts of area and surface area to draw conclusions about the relationship between the two measures. The student accurately calculates the surface areas of both boxes in arrangements X and Y and correctly compares these surface areas to the amount of wasted space in each box. The student clearly shows or explains why a box with less wasted space does not necessarily have a smaller surface area.

**1 point:** The student shows partial understanding of how to apply the concepts of area and surface area to draw conclusions about the relationship between the two measures. The student makes a calculation error in determining the surface area of one or both boxes but is still able to draw a correct conclusion about the relationship between surface area and the amount of wasted space inside each box. **OR** The student calculates the correct surface area of both boxes but draws an incorrect conclusion about the relationship between surface area and the amount of wasted space inside each box.

**0 points:** The student shows inconsistent understanding of how to apply the concepts of area and surface area to draw conclusions about the relationship between the two measures. The student does not identify the correct surface area of either box and does not show an understanding of the relationship between the surface areas of each box and the amount of wasted space inside each box.

**Part B**, Question 3: Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student shows a thorough understanding of how to apply the concepts of surface area to identify an arrangement with a smaller surface area than a given arrangement. The student provides an example of a box with a different arrangement than the one proposed by the company, correctly calculates the surface area of the different box, and shows that the surface area is less than the surface area of the box proposed by the company.

**1 point:** The student shows partial understanding of how to apply the concepts of surface area to identify an arrangement with a smaller surface area than a given arrangement. The student provides an example of a box with a different arrangement than the one proposed by the company, but makes a minor error calculating its surface area.

**0 points:** The student shows inconsistent understanding of how to apply the concepts of surface area to identify an arrangement with a smaller surface area than a given arrangement. The student shows an arrangement for a box that either does not have a smaller surface area than the one proposed by the company or the student does not understand how to calculate or compare the surface areas of both boxes.

**Part C**

Question 1: Correct responses for the surface area and volume will each receive 1 point and are scored independently.

Surface area:  $317.24 \text{ cm}^2$

Volume:  $410.54 \text{ cm}^3$

**Part C**, Question 2: Responses to this item will receive 0-5 points, based on the following:

**5 points:** The student shows a thorough understanding of how to use algebraic and geometric reasoning to find the dimensions of a cylinder with a given volume and minimized surface area. The student uses the relationship between the volume and surface area of a cylinder to determine a correct function for the surface area in terms of the radius. The student graphs and interprets this function correctly to identify its minimum  $x$ -value. The student identifies the correct radius, 4.03 cm, and calculates the correct height, 8.05 cm, of the cylinder with this minimum surface area. All student calculations and interpretations are performed without using any available hints.

**4 points:** The student shows a strong understanding of how to use algebraic and geometric reasoning to find the dimensions of a cylinder with a given volume and minimized surface area. The student uses the relationship between the volume and surface area of a cylinder to determine a correct function for the surface area in terms of the radius. The student graphs this function but does not interpret it correctly to find the correct radius, but calculates a correct height using the incorrect radius. Or the student graphs and interprets the graph to identify the correct radius but calculates the incorrect height. All student calculations and interpretations are performed without using any available hints.

**3 points:** The student shows an average understanding of how to use algebraic and geometric reasoning to find the dimensions of a cylinder with a given volume and minimized surface area. The student uses the first hint describing the relationship between the volume and surface area of a cylinder to determine a correct function for the surface area in terms of the radius. The student graphs and interprets this function correctly to identify its minimum  $x$ -value. The student identifies the correct radius and height of the cylinder with this minimum surface area.

**2 points:** The student shows partial understanding of how to use algebraic and geometric reasoning to find the dimensions of a cylinder with a given volume and minimized surface area. The student uses the first two hints describing the relationship between the volume and surface area of a cylinder and giving the student the function for the surface area in terms of the radius. The student graphs and interprets this function correctly to identify its minimum  $x$ -value. The student identifies the correct radius and height of the cylinder with this minimum surface area. **OR** The student uses the first hint

describing the relationship between the volume and surface area of a cylinder to determine a correct function for the surface area in terms of the radius. The student graphs and interprets this function incorrectly. As a result, the student identifies the incorrect radius and incorrect height of the cylinder with this minimum surface area. **OR** The student uses the first hint describing the relationship between the volume and surface area of a cylinder but makes a minor error in determining the function for the surface area in terms of the radius. The student graphs and interprets this function correctly to identify a minimum  $x$ -value. The student identifies an appropriate radius and height based on the incorrect function graph.

**1 point:** The student shows limited understanding of how to use algebraic and geometric reasoning to find the dimensions of a cylinder with a given volume and minimized surface area. The student uses all three hints to help determine the minimum surface area for the cylinder with the given volume. The student manipulates the spreadsheet to determine the correct value of the radius and calculates the correct height of the cylinder. **OR** The student uses the first two hints describing the relationship between the volume and surface area of a cylinder and giving the student the function for the surface area in terms of the radius. The student graphs this function correctly but identifies an incorrect, but close (within 0.5 cm), minimum value for the radius. The student identifies the correct height based on the incorrect radius.

**0 points:** The student shows inconsistent understanding of how to use algebraic and geometric reasoning to find the dimensions of a cylinder with a given volume and minimized surface area. The student uses all three hints to help determine the minimum surface area for the cylinder with the given volume. The student incorrectly manipulates the spreadsheet to determine an incorrect value of the radius and calculates an incorrect height of the cylinder. **OR** The student uses no hints and cannot determine the correct function for the surface area in terms of the radius. The student either doesn't graph the flawed function correctly or misinterprets how to find its minimum value. The student does not identify the correct radius or height. **OR** The student uses one or two hints but does not graph or interpret the function correctly to find its minimum value. The student does not identify the correct radius or height.

**Part C**, Question 3: Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student shows a thorough understanding of how to justify and support a valid recommendation using mathematical reasoning. The student provides a complete and accurate justification as to whether the standard can size should be changed. The student supports his or her reasoning using both the surface area and the amount of wasted space inside the packaging for both can sizes.

**1 point:** The student shows partial understanding of how to justify and support a valid recommendation using mathematical reasoning. The student provides an incomplete or partially accurate justification as to whether the standard can size should be changed. The

student supports his or her reasoning using either the surface area or the amount of wasted space inside the packaging for both can sizes.

**0 points:** The student shows an inconsistent understanding of how to justify and support a valid recommendation using mathematical reasoning. The student provides an incomplete and inaccurate justification as to whether the standard can size should be changed.

Scoring Rubric for **Part D**

**Part D, Question 1:** Responses to this item will receive 0-1 point, based on the following:

**1 point:** The student shows a thorough understanding of how to describe a method for collecting data to be used in a random sample. The student provides a reasonable explanation to indicate that the data collected would be random.

**0 points:** The student shows an inconsistent understanding of how to describe a method for collecting data to be used in a random sample. The student provides an explanation that does not indicate an understanding of what a random sample is.

**Part D, Question 2:** Responses to this item will receive 0-1 point, based on the following:

**1 point:** The student shows a thorough understanding of how to use an online graphing utility and given data to create a box plot. The output of the box plot is accurate.

**0 points:** The student shows an inconsistent understanding of how to use an online graphing utility and given data to create a box plot. The output of the box plot is missing, incomplete, or inaccurate.

**Part D, Question 3:** Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student shows a thorough understanding of how to justify and support a conjecture using mathematical reasoning. The student provides a complete and accurate justification as to whether the other beverage company is under-filling its cans. The student supports his or her reasoning using the box plot and the accompanying statistics from the box plot.

**1 point:** The student shows partial understanding of how to justify and support a conjecture using mathematical reasoning. The student provides an incomplete or partially accurate justification as to whether the other beverage company is under-filling its cans. The student supports his or her reasoning using minimal statistics from the box plot.

**0 points:** The student shows an inconsistent understanding of how to justify and support a conjecture using mathematical reasoning. The student provides an incomplete and inaccurate justification as to whether the other beverage company is under-filling its cans.

**MAT.HS.PT.4.HMOFC.A.268**

|                              |   |
|------------------------------|---|
| Sample Item ID:              | MAT.HS.PT.4.HMOFC.A.268   |
| Title:                       | Home Office   |
| Grade:                       | HS  |
| Primary Claim:               | <b>Claim 4: Modeling and Data Analysis</b><br>Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.   |
| Secondary Claim(s):          | Claim 1: Concepts and Procedures<br>Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.<br><br>Claim 2: Problem Solving<br>Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.   |
| Primary Content Domain       | <b>Number and Quantity</b>  |
| Secondary Content Domain(s): | Geometry, Functions, Algebra  |
| Assessment Target(s):        | 4 A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.<br><br>4 B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.<br><br>4 G: Identify, analyze, and synthesize relevant external resources to pose or solve problems.<br><br>4 D: Interpret results in the context of a situation.<br><br>1 C: Reason quantitatively and use units to solve problems.<br><br>1 G: Create equations that describe numbers or relationships.<br><br>1 E (Gr 7): Draw, construct, and describe geometrical figures and describe the relationships between them.<br><br>1 F (Gr 7): Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.<br><br>1 A (Gr 7): Analyze proportional relationships and use them to solve real-world and mathematical problems.<br><br>1 D (Gr 7): Solve real-life and mathematical problems using numerical and algebraic expressions and equations.<br><br>1 J (Gr 6): Summarize and describe distributions. |

|   |   |
|---|---|
|   | 2 B: Select and use appropriate tools strategically.  |
| Standard(s):  | N-Q.1, N-Q.3, A-CED.1, A-REI.4, F-IF.2, F-BF.2, F-LE.2, G-SRT.8, 7.G.1, 7.G.6, 7.RP.3, 7.EE.3, 6.SP.5   |
| Mathematical Practice(s):   | 1, 2, 3, 4, 5, 6, 7   |
| DOK:  | 3   |
| Item Type:  | PT  |
| Score Points:   | 16  |
| Difficulty:   | M   |
| How this task addresses the “sufficient evidence” for this claim: | The student uses concepts of geometry, functions, algebraic thinking, and number sense to accomplish tasks associated with having a home office built and identifying amounts that can be used as deductions based on the area of the home office. The work is supported by calculations and explanations of reasoning.   |
| Target-specific attributes (e.g., accessibility issues):          | Accommodations may be necessary for students who have vision challenges, fine motor-skill challenges, and language-processing challenges.   |
| Stimulus/Source:  | Sources used for flooring prices in Flooring Options table:<br><a href="http://www.homedepot.com/webapp/catalog/servlet/Search?keyword=carpet+prices&amp;selectedCatgry=SEARCH+ALL&amp;langId=-1&amp;storeId=10051&amp;catalogId=10053&amp;Ns=None&amp;Ntpr=1&amp;Ntpc=1#/c=1&amp;rpp=96">http://www.homedepot.com/webapp/catalog/servlet/Search?keyword=carpet+prices&amp;selectedCatgry=SEARCH+ALL&amp;langId=-1&amp;storeId=10051&amp;catalogId=10053&amp;Ns=None&amp;Ntpr=1&amp;Ntpc=1#/c=1&amp;rpp=96</a><br><br><a href="http://www.homedepot.com/webapp/catalog/servlet/Search?keyword=tile+prices&amp;selectedCatgry=SEARCH+ALL&amp;langId=-1&amp;storeId=10051&amp;catalogId=10053&amp;Ns=None&amp;Ntpr=1&amp;Ntpc=1">http://www.homedepot.com/webapp/catalog/servlet/Search?keyword=tile+prices&amp;selectedCatgry=SEARCH+ALL&amp;langId=-1&amp;storeId=10051&amp;catalogId=10053&amp;Ns=None&amp;Ntpr=1&amp;Ntpc=1</a><br><br><a href="http://www.homedepot.com/webapp/catalog/servlet/Search?keyword=laminate+prices&amp;selectedCatgry=SEARCH+ALL&amp;langId=-1&amp;storeId=10051&amp;catalogId=10053&amp;Ns=None&amp;Ntpr=1&amp;Ntpc=1">http://www.homedepot.com/webapp/catalog/servlet/Search?keyword=laminate+prices&amp;selectedCatgry=SEARCH+ALL&amp;langId=-1&amp;storeId=10051&amp;catalogId=10053&amp;Ns=None&amp;Ntpr=1&amp;Ntpc=1</a><br><br>Source used for tax details:<br><a href="http://www.irs.gov/newsroom/article/0,,id=108138,00.html">http://www.irs.gov/newsroom/article/0,,id=108138,00.html</a> |
| Notes:  | Multi-Part Task   |
| Task Overview:  | Students will calculate the area of a home office space in a finished basement, given a set budget amount. Then students will decide on a type of flooring to use for the home office given a set of flooring options and then calculate its cost. They will relate the area of the home office to utility expenses for the entire house to predict a tax deduction amount for the use of the home office.<br><br>All parts, A through D, will be scored for this task.   |
| Teacher preparation/Resource requirements:                        | Teacher preparation:<br>At least a day or two prior to starting this task, the teacher should put together a “Flooring Options” table which shows some different types of flooring and at least three sample costs and related sizes or measured units for each of these options. The table below can be used until these costs are no longer viable (outdated). This table is the same as the one used in the  |



|   |   |
|---|---|
|   | <p>Sample Top-Score Response. This table will be given to students to use in <i>Part B</i> of the task. The table should include costs given in different units (e.g., cost per sq ft vs. sq yd) to have the student make use of conversion skills. Point this out to students so they are aware that conversions will be necessary.</p> <p>Resource requirements:<br/>                 Spreadsheet software and graphing paper for calculation work and diagram manipulation/work must be available to all students. Calculators should be available to students, either online or physically.</p> |
| Teacher Responsibilities During Administration: | Monitor individual student work and facilitate class discussion at the beginning of Session 1. Provide resources as necessary.  |
| Time Requirements:                              | Two sessions totaling no more than 120 minutes; <i>Parts A</i> and <i>B</i> will be completed during Session 1. <i>Parts C</i> and <i>D</i> will be completed during Session 2. All parts during both sessions will be performed individually.  |

Prior to actually starting this task, the teacher should lead a five-minute class discussion about different types of flooring that can be used to convert a basement floor into a home office. The discussion should center on what types of flooring are available and characteristics of each flooring type.

The teacher should then distribute copies of the “Flooring Options” table shown below. Explain to the class that this table has the three flooring types they will consider for the home office. This will be used in their response to *Part B*. Point out that the pricing in the table is not always in the same units and that students will need to consider this when using the table in *Part B*.

### Flooring Options

| Type     | Option 1                                     | Option 2                                | Option 3                                |
|----------|--|---|---|
| Carpet   | \$31.98 for box of carpet tiles (20 sq. ft.) | \$8.73 sq. yd.                          | \$22.23 sq. yd.                         |
| Tile     | 16 in. × 16 in. tiles<br>\$6.49 sq. ft.      | 12 in. × 12 in. tiles<br>\$1.48 sq. ft. | 12 in. × 12 in. tiles<br>\$2.89 sq. ft. |
| Laminate | \$8.91 sq. yd.                               | \$1.69 sq. ft.                          | \$3.99 sq. ft.                          |

## Session 1

### Home Office

You want to finish your basement and use it as a home office. You plan to hire a contractor to do the work. The amount charged by the contractor is based on the area of the room to be finished. The amount, however, does not include the cost of flooring.

In this task, you will use a blueprint of the basement. You need to decide what area of the basement will be finished by the contractor. You also need to decide on the type of flooring that will go in the basement and price it so that you stay within your budget. Once you know the area of the home office, you will use that information to help determine how it will affect your taxes for the next year.

To accomplish this, you will do the following:

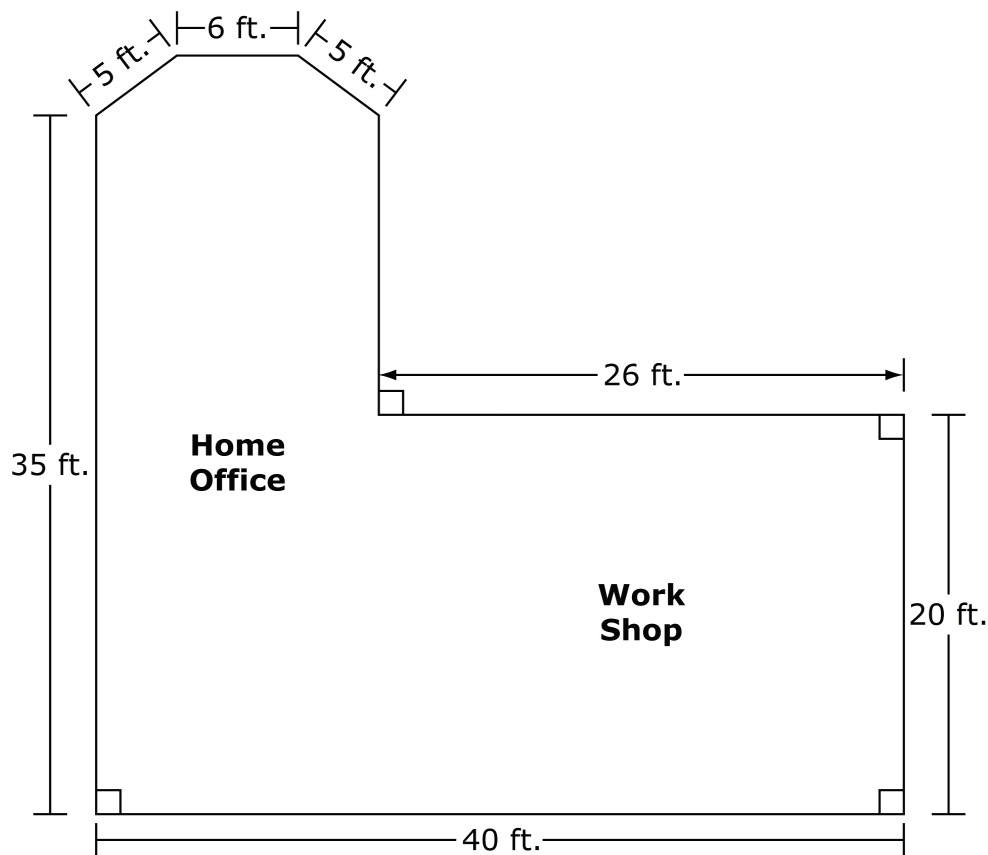
1. Find the maximum area of the basement that can be finished based on the rate charged by the contractor.
2. Decide on the type of flooring you will use given a "Flooring Options" table and calculate the cost of the flooring.
3. Calculate the area used for the actual home business, excluding the area of a bathroom.
4. Calculate expected tax deductions for converting part of the basement as a home office using the following:
  - the area of the home office
  - the area of the entire house
  - the past year's utility expenses

**Part A**

**Determine the Area  
of the Finished Basement**

You want part of your basement converted into a home office. You will hire a contractor to do this work. Your budget for the contractor is \$30,000. The contractor charges \$50 per square foot to finish the basement for the home office. The finished work includes everything but the cost of flooring.

The diagram below represents the blueprint of your basement.



Only the left side of the basement will be finished for the home office. The workshop area will remain unfinished. You want the

greatest possible area for the home office based on your contractor budget.

1. Determine the area of the basement that will be used for the home office. This involves separating the home office area from the workshop area. To do this, draw a vertical line between the two areas directly on the blueprint. The left side of the blueprint will represent the area of the home office. The right side will represent the area of the workshop.

[Use the partition or single line segment TE template or ruler tool.]

Explain how you decided where to draw the line that separates the two areas. In your explanation, give the dimensions of the rectangular workshop.

## **Part B**

### **Flooring**

Use the “Flooring Options” table to help you choose a type of flooring for the finished basement. You will use the average price of that flooring type to calculate your flooring cost.

Base your decision on the following criteria:

- Flooring options in the table
- Average cost of each flooring option
- A flooring budget of \$2000

To find the average cost of each flooring option, use the mean costs of those given in the table. If costs are not in the same units, do the conversions necessary to change them to the same units.

2. What type of flooring did you decide to use? Explain whether or not it was necessary to pick the least expensive flooring option in order to stay within your budget.

3. Based on the flooring budget and the average cost of the flooring you chose, find the total cost of flooring for the finished basement. Show or explain how you found your answer.

**End of Session 1**

## Session 2

### *Part C*

#### **Including a Bathroom**

You decide to include a bathroom as part of your home office. You ask the contractor to allow 10% of the finished portion of the basement to be used for the bathroom. You also want the length of the bathroom to be 3 feet longer than the width.

4. Write a polynomial equation that can be used to find the dimensions of the bathroom. Then determine the dimensions of the bathroom. Round each dimension to the nearest half-foot.

### *Part D*

#### **Home Office Expenses and Tax Deductions**

When you use a home office, a percentage of total home utility expenses can be deducted from your taxes. This is based on the percentage of your house that is occupied by the home office. When the finished basement is complete, the total square footage of the entire house will include the home office portion of the finished basement but not the bathroom. The area used as the home office will also be used as part of the tax calculation for what you owe to the government.

You want to estimate the amount of utility expenses that can be deducted from next year's taxes, when your home office is complete. Below is a spreadsheet which lists all of your utility expenses for the past year.

| <b>Utility Expenses</b> |                 |                 |                    |                 |
|-------------------------|-----------------|-----------------|--------------------|-----------------|
| <b>Month</b>            | <b>Phone</b>    | <b>Heat</b>     | <b>Electricity</b> | <b>Trash</b>    |
| Jan                     | \$61.62         | \$95.47         | \$152.54           | \$316.00        |
| Feb                     | \$62.71         | \$154.83        | \$164.89           |                 |
| Mar                     | \$57.05         | \$124.80        | \$125.29           |                 |
| Apr                     | \$67.48         | \$75.57         | \$130.03           | <b>Sewer</b>    |
| May                     | \$66.47         | \$33.80         | \$192.15           | \$94.94         |
| Jun                     | \$56.23         | \$32.30         | \$235.05           |                 |
| Jul                     | \$50.13         | \$35.22         | \$254.02           |                 |
| Aug                     | \$78.52         | \$36.42         | \$250.13           | <b>Water</b>    |
| Sep                     | \$66.33         | \$31.41         | \$240.34           | \$178.33        |
| Oct                     | \$57.73         | \$31.38         | \$177.52           |                 |
| Nov                     | \$57.97         | \$54.06         | \$229.53           |                 |
| Dec                     | \$61.52         | \$172.54        | \$196.26           |                 |
|                         |                 |                 |                    |                 |
| <b>Totals:</b>          | <b>\$743.76</b> | <b>\$877.80</b> | <b>\$2,347.75</b>  | <b>\$589.27</b> |



A percentage of these total expenses can be applied as a tax deduction. However, the phone expenses include some personal calls that cannot be deducted. Only the monthly phone fee and your business-related calls can be deducted.

Your phone bill contains these charges:

- \$36 monthly fee for using the phone
- \$0.12 per minute for each long-distance call made

Your business-related calls are all long-distance calls. You kept a record of the number of minutes you were charged for business-related calls last year. The spreadsheet below shows these data by month.

| <b>Business Calls</b> |                          |                       |
|-----------------------|--------------------------|-----------------------|
| <b>Month</b>          | <b>Number of Minutes</b> | <b>Phone Expenses</b> |
| Jan                   | 96                       |                       |
| Feb                   | 68                       |                       |
| Mar                   | 90                       |                       |
| Apr                   | 165                      |                       |
| May                   | 129                      |                       |
| Jun                   | 99                       |                       |
| Jul                   | 48                       |                       |
| Aug                   | 204                      |                       |
| Sep                   | 155                      |                       |
| Oct                   | 67                       |                       |
| Nov                   | 80                       |                       |
| Dec                   | 52                       |                       |
|                       |                          |                       |
| <b>Total:</b>         |                          |                       |

[Note: Students should be able to use the above spreadsheet to enter formulas and calculate costs in any of the empty cells.]

5. Determine a function to find the total phone expense that can be applied toward your home office tax deduction. Apply the function in the spreadsheet to calculate each month's phone expense that applies toward the tax deduction. Find the total applicable phone expense. Use the labeled column in the spreadsheet to show these amounts.

You are now ready to estimate the total amount of utility expenses that can be applied toward your tax deduction next year. Your estimate should account for the following:

- The square footage of your house before adding the home office is 1850 square feet.
  - The areas of bathrooms are not included in the square footage of the house.
  - Deductions are based on the total of last year's utility expenses. These include heat, electricity, trash, water, sewer, and business-related phone expenses.
  - The expected increases in utility expenses are due to the use of the home office.
6. Determine a reasonable estimate for the amount of the tax deduction for next year's taxes. Show or explain how you determined this estimate. You may use a combination of diagrams, mathematical equations or formulas, and words.

**End of Session 2**

*Sample Top-Score Response:*

Sample table prepared by teacher:

**Flooring Options**

| Type     | Option 1                                     | Option 2                                | Option 3                                |
|----------|--|---|---|
| Carpet   | \$31.98 for box of carpet tiles (20 sq. ft.) | \$8.73 sq. yd.                          | \$22.23 sq. yd.                         |
| Tile     | 16 in. × 16 in. tiles<br>\$6.49 sq. ft.      | 12 in. × 12 in. tiles<br>\$1.48 sq. ft. | 12 in. × 12 in. tiles<br>\$2.89 sq. ft. |
| Laminate | \$8.91 sq. yd.                               | \$1.69 sq. ft.                          | \$3.99 sq. ft.                          |

**Part A**

1. The student should provide a complete and correct explanation of how the calculation for the area of the home office was made. The separator line should be marked so that the area of the workshop is 22 feet long and 20 feet wide.

First I calculated the largest possible area of the home office based on the contractor budget.

Largest possible area of home office:  $\$30,000 \div \$50 \text{ per sq ft} = 600 \text{ total sq ft}$

Then I divided the blueprint into smaller shapes and found the areas of those shapes. The top trapezoid is made up of two 3-4-5 right triangles and one 3x6 rectangle.

Its height is 3 feet, so its area is  $\frac{1}{2}(6 + 14)(3) = 30 \text{ sq ft}$ .

The area of the large rectangle below the trapezoid is  $(14)(35) = 490 \text{ sq ft}$ .

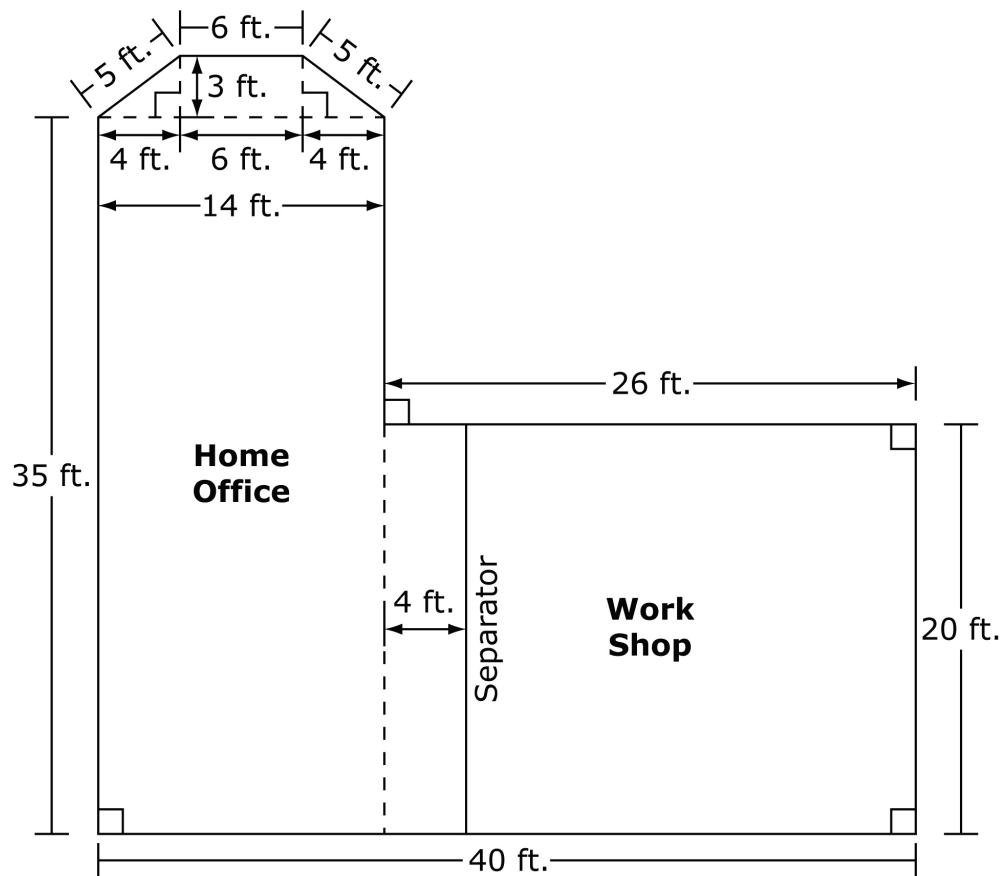
To see how far I must go into the section near the workshop, I need to solve this equation:

$$600 = 30 + 490 + (20x)$$

$$600 - 520 = 20x$$

$$x = 4$$

The home office extends 4 feet toward the workshop to give it a total area of 600 sq ft.



The width of the workshop is 20 feet. The length of the workshop is  $26 - 4 = 22$  feet. Its dimensions are 20 ft. by 22 ft. Its area is  $(20)(22) = 440$  sq ft.

**Part B**

2. The student should choose one type of flooring for the home office from the “Flooring Options” table. The student should compare the average flooring costs in the table. The average costs should reflect any converted costs if the units in their pricing were not the same. The student should explain how the cost relates to the budget and whether or not he/she had to choose the least expensive flooring option in order to stay within his or her budget.

The average cost of each flooring type is:

Carpeting average:

Option 1:  $\$31.98 \div 20 = \$1.60$  sq ft

Option 2:  $\$8.73 \text{ sq yd} \times \frac{1 \text{ sq yd}}{9 \text{ sq ft}} = \$0.97$  sq ft

Option 3:  $\$22.23 \text{ sq yd} \times \frac{1 \text{ sq yd}}{9 \text{ sq ft}} = \$2.47$  sq ft

Average =  $(\$1.60 + \$0.97 + \$2.47) \div 3 = \$1.68$  sq ft

Tile average:

Option 1: \$6.49 sq ft

Option 2: \$1.48 sq ft

Option 3: \$2.89 sq ft

Average =  $(\$6.49 + \$1.48 + \$2.89) \div 3 = \$3.62$  sq ft

Laminate average:

Option 1:  $\$8.91 \text{ sq yd} \times \frac{1 \text{ sq yd}}{9 \text{ sq ft}} = \$0.99$  sq ft

Option 2: \$1.69 sq ft

Option 3: \$3.99 sq ft

Average =  $(\$0.99 + \$1.69 + \$3.99) \div 3 = \$2.22$  sq ft

The flooring budget, in cost per sq ft, is  $\$2000 \div 600 \text{ sq ft} = \$3.33$  sq ft.

Tile costs too much for the given budget, so I chose the laminate flooring. The laminate costs more than carpeting, on average, but the total cost still fits within the budget.

3. The student should calculate the cost of flooring based on the square footage determined in *Part A* and the average cost of the flooring determined in question 2. The calculation should include any conversions of flooring costs to square feet. The total cost should also remain within the budget amount of \$2000.

Budget: \$2000

Average cost per sq ft of laminate: \$2.22 sq ft

Area of flooring: 600 sq ft

Cost of flooring =  $600 \times \$2.22 = \$1332$

### **Part C**

4. The student writes the correct polynomial equation.

$$w(w + 3) = 600(10\%)$$

$$w^2 + 3w = 60$$

$$w^2 + 3w - 60 = 0$$

The student correctly solves the equation for  $w$  using the quadratic formula and rounds the answer to the nearest half-foot.

$$w = \frac{-3 \pm \sqrt{3^2 - 4(1)(-60)}}{2(1)} = \frac{-3 \pm \sqrt{9 + 240}}{2} \approx \frac{-3 \pm 15.7797}{2}$$

$$w \approx 6.39 \text{ or } -9.39$$

Since widths cannot be negative, the width  $\approx 6.39$  ft which, to the nearest half-foot, rounds to 6.5 ft. The length to the nearest half-foot =  $6.5 + 3 = 9.5$  ft.

### **Part D**

5. The student correctly determines a function for the phone expense,  $y = 0.12x + 36$ , and

applies it to the data in the spreadsheet. The student writes the function as a formula in one of the empty columns of the spreadsheet and finds the sum of all 12 months of resulting data.

| Business Calls |                   |                 |
|----------------|-------------------|-----------------|
| Month          | Number of Minutes | Phone Expenses  |
| Jan            | 96                | \$47.52         |
| Feb            | 68                | \$44.16         |
| Mar            | 90                | \$46.80         |
| Apr            | 165               | \$55.80         |
| May            | 129               | \$51.48         |
| Jun            | 99                | \$47.88         |
| Jul            | 48                | \$41.76         |
| Aug            | 204               | \$60.48         |
| Sep            | 155               | \$54.60         |
| Oct            | 67                | \$44.04         |
| Nov            | 80                | \$45.60         |
| Dec            | 52                | \$42.24         |
|                |                   |                 |
| <b>Total:</b>  |                   | <b>\$582.36</b> |

6. Student work and explanations should include the following:

The student finds the correct area of the home office, not including the bathroom.

Bathroom area is 10% of the home office area =  $600 \times 10\% = 60$  sq ft.

Area of home office, excluding bathroom =  $600 - 60 = 540$  sq ft

The student finds the correct percent of square footage of the home office.

$540 \div (1850 + 540) \approx 0.2259$  or 22.59%. To the nearest percent, the home office is 23% of the total square footage.

The student finds the correct applicable utility expenses from last year's data.

heat + electricity + trash + sewer + water + phone

$$877.80 + 2347.75 + 316 + 94.94 + 178.33 + 582.36 = \$4397.18$$

The student reasonably identifies utility expenses that are likely to increase and estimates the increase in expenses due to the use of the home office.

Not all utility expenses will change. Certain ones are not affected by an increase in the square footage of a house, such as phone, trash, sewer, and water expenses. Others will increase, however, with additional square footage. The utilities that are expected to increase are heat and electricity, since the home office will need to be climate-controlled and powered for light and business equipment.

To estimate the amount that heat and electricity expenses are expected to increase, I'll estimate the amount of these utility expenses for 540 square feet of the existing house. Then I'll add that increased amount to the total expenses.

$$(540 \div 1850)(877.80 + 2347.75) = \$941.51$$

$$\text{Total estimated expenses with the home office} = 4397.18 + 941.51 = \$5338.69$$

The amount of the tax deduction =

$$\text{percent of total area occupied by home office} \times \text{total estimated expenses with home office}$$
$$23\% \times 5338.69 = \mathbf{\$1227.90}$$

*Scoring Notes:*

Class discussion prior to starting the task will not be scored. Each question in *Parts A* through *D* is evaluated individually. The total number of points is determined by adding the points assigned for each question.

*Scoring Rubric:*

Scoring Rubric for **Part A:**

Question 1: Responses to this item will receive 0–4 points, based on the following:

**4 points:** The student shows a thorough understanding of how to use geometric and algebraic concepts to determine area. The student correctly identifies where to place the partition line in the basement. The student correctly calculates areas for sections of the home office using the Pythagorean theorem and algebraic equations. The student correctly determines the dimensions of the workshop area.

**3 points:** The student shows a strong understanding of how to use geometric and algebraic concepts to determine area. The student correctly identifies where to place the partition line in the basement. The student correctly calculates areas for sections of the home office using the Pythagorean theorem and algebraic equations. However, the student either forgets to determine the dimensions of the workshop or calculates incorrect dimensions. **OR** The student correctly identifies where to place the partition line in the basement and correctly determines the dimension of the workshop area. However, the student's work or explanation is incomplete.

**2 points:** The student shows partial understanding of how to use geometric and algebraic

concepts to determine area. The student makes one or two calculation errors for the area used for the home office. As a result, the student identifies a partition line in the basement close to, but not exactly, where it should go. The student determines the correct dimensions of the workshop based on the incorrect partition line.

**1 point:** The student shows a limited understanding of how to use geometric and algebraic concepts to determine area. The student makes some appropriate area calculations to help determine where to place the partition line but does not identify the correct location for the partition. The student does not determine the correct dimensions of the workshop.

**0 points:** The student shows inconsistent understanding of how to use geometric and algebraic concepts to determine area. The student's work contains many calculation errors, a missing or incorrect partition line, and missing or incorrect dimensions of the workshop.

#### Scoring Rubric for *Part B*:

Question 2: Responses to this item will receive 0–2 points, based on the following:

**2 points:** The student shows a thorough understanding of how to analyze real-world scenarios and make productive use of knowledge to make decisions and solve problems. The student finds the correct average costs of each flooring type from the "Flooring Options" table, including any necessary conversions. The student refers to staying within the budget and whether or not he/she needed to choose the least expensive flooring option.

**1 point:** The student shows partial understanding of how to analyze real-world scenarios and make productive use of knowledge to make decisions and solve problems. The student finds the average costs of each flooring type from the "Flooring Options" table, including any necessary conversions. However, the student does not refer to the budget as a reason for choosing the type of flooring or whether or not he/she needed to choose the least expensive flooring option. **OR** The student makes one or two errors calculating the average costs of each flooring type from the "Flooring Options" table, including errors in any necessary conversions. However, the student stays within the budget and refers to whether or not he/she needed to choose the least expensive flooring option.

**0 points:** The student shows inconsistent understanding of how to analyze real-world scenarios and make productive use of knowledge to make decisions and solve problems. The student incorrectly calculates the average costs of one or more flooring types and does not stay within the budget or refer to whether or not he/she needed to choose the least expensive flooring option.

Question 3: Responses to this item will receive 0–1 point, based on the following:

**1 point:** The student shows a thorough understanding of how to solve problems in applied math. The student correctly calculates the cost of the flooring based on the cost of the flooring and square footage of the area. The student makes any necessary conversions and stays within the specified budget.

**0 points:** The student shows inconsistent understanding of how to solve problems in



applied math. The student incorrectly calculates the cost of the flooring. The student does not make necessary conversions and/or does not stay within the specified budget.

Scoring Rubric for **Part C**:

Question 4: Responses to this item will receive 0–3 points, based on the following:

**3 points:** The student shows a thorough understanding of how to apply algebraic concepts to solve problems in applied math. The student correctly writes and solves a polynomial equation to find the dimensions of the bathroom, rounded to the nearest half-foot.

**2 points:** The student shows some understanding of how to apply algebraic concepts to solve problems in applied math. The student correctly writes a polynomial equation but makes a minor error applying the quadratic formula.

**1 point:** The student shows partial understanding of how to apply algebraic concepts to solve problems in applied math. The student correctly writes a polynomial equation but makes one or two errors solving the equation. The student also does not round the dimensions to the nearest half-foot. **OR** The student incorrectly writes a polynomial equation but solves that equation correctly for the dimensions of the bathroom, rounded to the nearest half-foot.

**0 points:** The student shows inconsistent understanding of how to apply algebraic concepts to solve problems in applied math. The student does not correctly write or solve a quadratic equation to find the dimensions of the bathroom.

Scoring Rubric for **Part D**:

Question 5: Responses to this item will receive 0–2 points, based on the following:

**2 points:** The student shows a thorough understanding of how to apply an algebraic function and spreadsheet technology to solve problems in applied math. The student enters a correct function into the spreadsheet and determines the total deductible phone expenses.

**1 point:** The student shows partial understanding of how to apply an algebraic function and spreadsheet technology to solve problems in applied math. The student enters into the spreadsheet a function with the slope and the  $y$ -intercept reversed. However, the student applies this formula to each month and sums the amounts to determine the total. **OR** The student enters a correct function into the spreadsheet to determine each month's deductible phone expense but does not find the total sum.

**0 points:** The student shows inconsistent understanding of how to apply an algebraic function and spreadsheet technology to solve problems in applied math. The student enters an incorrect function into the spreadsheet unrelated to reversing the slope and  $y$ -intercept.

Question 6: Responses to this item will receive 0–4 points, based on the following:

**4 points:** The student shows a thorough understanding of how to analyze complex, real-world scenarios and construct mathematical models to solve problems. The student

calculates the correct percent of square footage occupied by the home office, excluding the area of the bathroom. The student determines the correct applicable utility expenses from the prior year. The student provides a reasonable explanation as to which utilities might increase with the addition of the home office and determines a reasonable estimate for those increased amounts. The student determines a reasonable estimate of the amount of the tax deduction based on the percentage of home office square footage.

**3 points:** The student shows a strong understanding of how to analyze complex, real-world scenarios and construct mathematical models to solve problems. The student correctly determines most of the calculations and/or estimates needed to assess the amount of the tax deduction for utility expenses.

**2 points:** The student shows partial understanding of how to analyze complex, real-world scenarios and construct mathematical models to solve problems. The student correctly determines some of the calculations and/or estimates needed to assess the amount of the tax deduction for utility expenses.

**1 point:** The student shows a limited understanding of how to analyze complex, real-world scenarios and construct mathematical models to solve problems. The student correctly determines one of the calculations and/or estimates needed to assess the amount of the tax deduction for utility expenses.

**0 points:** The student shows inconsistent understanding of how to analyze complex, real-world scenarios and construct mathematical models to solve problems. The student does not correctly determine any of the calculations or estimates needed to assess the amount of the tax deduction for utility expenses.

**MAT.HS.PT.4.TUITN.A.298**

|                              |  |
|------------------------------|--|
| Sample Item ID:              | MAT.HS.PT.4.TUITN.A.298  |
| Title:                       | College Tuition  |
| Grade:                       | HS   |
| Primary Claim:               | <b>Claim 4: Modeling and Data Analysis</b><br>Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.  |
| Secondary Claim(s):          | <p>Claim 1: Concepts and Procedures<br/>Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.</p> <p>Claim 2: Problem Solving<br/>Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.</p> <p>Claim 3: Communicating Reasoning<br/>Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.</p>   |
| Primary Content Domain:      | <b>Statistics and Probability</b>  |
| Secondary Content Domain(s): | Functions, Algebra, Number and Quantity  |
| Assessment Target(s):        | <p>4 A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.</p> <p>4 E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.</p> <p>4 G: Identify, analyze, and synthesize relevant external resources to pose or solve problems.</p> <p>4 B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.</p> <p>4 F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).</p> <p>4 D: Interpret results in the context of a situation.</p> <p>1 G: Create equations that describe numbers or relationships.</p> <p>1 P: Summarize, represent, and interpret data on a single-count or measurement variable.</p> <p>2 B: Select and use appropriate tools strategically.</p> <p>3 F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions.</p> |

HS Mathematics Sample PT Form Claim 4



|   |  |
|---|--|
| Standard(s):  | S-ID.6, S-ID.7, S-ID.3, S-ID.2, S-ID.8, F-LE.1, F-LE.2, F-LE.5, F-IF.2, A-CED.2, N-Q.3, 8-SP.1, 8.SP.2, 7.RP.3, 7.EE.3   |
| Mathematical Practice(s):   | 1, 2, 4, 5, 6, 7   |
| DOK:  | 4  |
| Item Type:  | PT   |
| Score Points:   | 14   |
| Difficulty:   | M  |
| How this task addresses the “sufficient evidence” for this claim: | The student uses concepts of statistics, functions, and algebraic thinking to accomplish tasks associated with predicting the future costs of college tuition. The work is supported by calculations and explanations of reasoning.  |
| Target-specific attributes (e.g., accessibility issues):          | Accommodations may be necessary for students who have vision challenges, fine-motor-skills challenges, and language-processing challenges.   |
| Stimulus/Source:  | <p>For articles used in prework:</p> <p>Article 1<br/> <a href="http://www.usatoday.com/news/education/story/2011-10-25/public-college-costs-increase/50919598/1">http://www.usatoday.com/news/education/story/2011-10-25/public-college-costs-increase/50919598/1</a></p> <p>Article 2<br/> <a href="http://www.insidehighered.com/news/2010/10/28/tuition#ixzz1mbLlzzuN">http://www.insidehighered.com/news/2010/10/28/tuition#ixzz1mbLlzzuN</a></p> <p>Article 3<br/> <a href="http://www.changinghighereducation.com/2012/01/so-let-me-put-colleges-and-universities-on-notice-if-you-cant-stop-tuition-from-going-up-the-funding-you-get-from-taxpay.html">http://www.changinghighereducation.com/2012/01/so-let-me-put-colleges-and-universities-on-notice-if-you-cant-stop-tuition-from-going-up-the-funding-you-get-from-taxpay.html</a></p> <p>To be used in conjunction with prework research:<br/> <a href="http://chronicle.com/article/Interactive-Tool-Tuition-Over/125043/">http://chronicle.com/article/Interactive-Tool-Tuition-Over/125043/</a></p> <p>A simulated search will be developed in a similar fashion to the search tool provided on this Web site. The search tool will contain a subset of the data on this site. That subset of data will be from the collection of schools/institutions each student chooses in the days leading up to this activity.</p> <p>For data on average college tuition and fees:<br/> <a href="http://nces.ed.gov/programs/digest/d10/tables/dt10_345.asp">http://nces.ed.gov/programs/digest/d10/tables/dt10_345.asp</a></p> |
| Notes:  | Multi-part task  |
| Task Overview:  | <p>Students will research data on college tuition over time. They will analyze their data in groups and individually to develop a model that best fits their collected data. Their models will then be used to predict future costs of college tuition.</p> <p><i>Parts C, D, and E</i> will be the only scored portions of this task.</p>   |
| Teacher preparation / Resource requirements:                      | <p>Teacher preparation:<br/>                     Up to 3 – 5 days prior to the administration of this task, students will be assigned a prework task that will be used to gather data in <i>Part A</i> of the task and to compare data in <i>Part C</i></p>  |

|  |   |
|--|---|
|  | <p>of the task. The prework should be done individually, outside of school, and given to the teacher at least one day before the start of the task. The teacher will compile data for a simulated search based on the prework information. The simulated search will be performed individually during <i>Part A</i> of this task. In the latter part of Session 1, <i>Part B</i> will incorporate group work to analyze data and will require the teacher to coordinate partner/group work for this part of the task. The remaining parts of this task will be completed independently. Session 2 will involve modeling and interpreting the data analyzed during the group work.</p> <p>Resource requirements:<br/>Spreadsheet software and computers must be available to all students, as well as research tools to help students compile simulated data. Calculators should be available to students, either online or physically. Copies of a specific news article will be handed out and read as part of the prework activity.</p> |
| <p>Teacher Responsibilities During Administration:</p> | <p>Monitor individual student work and monitor group work.<br/>Provide resources as necessary.</p>  |
| <p>Time Requirements:</p>                              | <p>Two sessions totaling no more than 120 minutes. <i>Parts A</i> and <i>B</i> will be completed during Session 1. <i>Part A</i> should be performed individually and <i>Part B</i> should be performed in small groups. <i>Parts C, D,</i> and <i>E</i> will be completed during Session 2. All tasks during Session 2 will be performed individually.</p>   |

**Prework:**

In preparation for this task, teachers must assign students the following task as an individual activity at least 3 days prior to the administration of the performance task. Teachers must hand out copies of these three articles to each student for this prework portion:

1. Marklein, Mary Beth (2011), "Tuition and fees rise more than 8% at U.S. public colleges," from *USA Today*, October 26.

Online articles:

2. Jaschik, Scott (2010), "Tuition Hikes of the Downturn," from *Inside Higher Ed*, October 28.

3. (2012), "The State of the Union on college costs," from *Changing Higher Education*, January 30.

[Note: A copy of each article is at the end of this task.]

**Teacher says:** Most students have plans to attend college after graduating from high school. There are many costs to consider when planning for college. The major costs are tuition and school-related fees, which are typically combined into one dollar amount. The cost of a college education is expected to increase from year to year. As a result, the yearly cost for a college education during a student's first year may be significantly different four years later when the student is ready to graduate.

During the two sessions of the upcoming performance task, you will be predicting the total costs for tuition and school-related fees, as a combined cost (which will be referred to as “tuition” throughout the task), for a college of your choice. Your assignment will include the following:

- Choose a college or university that you will use to predict the future cost of tuition. This can either be a local 2-year or 4-year institution or one that you would like to attend in the future. You must provide the name of the school and the type of institution (i.e., 2-year or 4-year, public or private, college or university).
- Find out what the current year’s tuition, including school-related fees, are for the school you chose. This information can often be found by calling the school’s admissions office, obtaining a current school catalog, or doing an Internet search. Be sure to get the cost for in-state students if the school is located in this state. Get the cost for out-of-state students if the school is not located in our state.
- Read the news articles “Tuition and fees rise more than 8% at U.S. public colleges,” “Tuition Hikes of the Downturn,” and “The State of the Union on college costs.”  
[Note: Teacher must distribute copies of each of these articles.]
- Use the information you obtained about the current year’s tuition at the school you chose and the information you read in the “Tuition and fees rise more than 8% at U.S. public colleges” news article to predict the cost of college tuition at your choice of schools the year you are first eligible to attend college. You should also predict the total tuition amount for the entire college education. This will be 2 years or 4 years based on the type of school you choose. Have your calculations and an explanation of how you determined your total predicted amount ready when the performance task officially begins.

*[Assumptions: The method of handling the research part of the prework is based on this item writer not fully knowing what tools will be available to teachers and students as they perform this task. It is based on the assumption that students will not be allowed to do online searches in the classroom and that some sort of simulated search will need to be developed. The description below is only a suggested possibility. There may very well be an easier way to handle this research portion, such as reproducing the site or using the site itself, if possible.]*

The prework from the first bullet (choosing a school) will be provided to teachers 4-5 school days before the start of the task. This is to give time for teachers to prepare a full list of schools that will be combined into one collective simulated search that all students will use

at the start of the performance task.

To do this, teachers will compile the complete list of schools provided by the students. They will enter each school into the search tool provided below.

<http://chronicle.com/article/Interactive-Tool-Tuition-Over/125043/>

Screen shots will be made of each school's tuition data. The screen shots will then be combined to form the simulated search tool to be used by the students when they begin Session 1 of the task.

[Notes: Ideally, the simulated search should be laid out in a similar fashion to the search tool provided on this Web site listed below. The simulated search will be a subset of all schools found on this site.

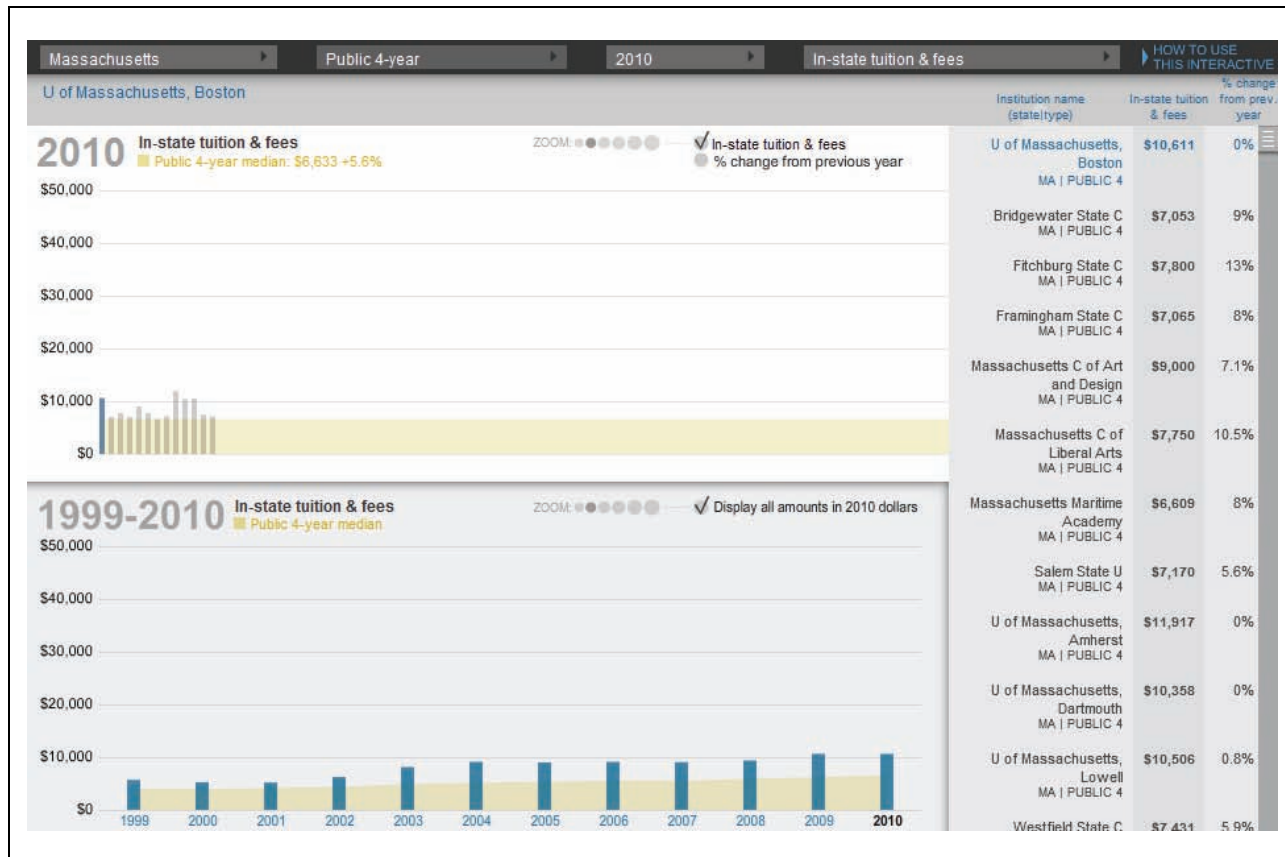
<http://chronicle.com/article/Interactive-Tool-Tuition-Over/125043/>

A sample screen shot is below. (It can be enlarged.) This is the school whose data are used in the sample response.

A simulated search should be created to have only the data from the collective list of schools provided by the students. Students will locate the school they specifically chose, and ideally have these two tables displayed: "1999-2010 In-state tuition & fees" and "1999-2010 Out-of-state tuition & fees."

Additional information:

1. If the simulated search can be computer based, scrolling over the bars from the bar graph will list both in- and out-of-state tuition and fees for the particular school.
2. If the simulation will not be available via computer, the output should be adjusted from what is shown in the screen shot below. (The screen shot below shows "2010 in-state tuition & fees" displayed on top, followed by the "in-state tuition and fees" table.) The output should be adjusted to show the "1999-2010 In-state tuition and fees" on top and the "1999-2010 Out-of-state tuition and fees" underneath.
3. The data on this Web site is based on tuition and fees only. No additional expenses are reflected unless specifically noted.
4. This is the data students will use in *Part A* of the task.]



## Session 1

[Note: Session 1 of the performance task consists of two *Parts*: *A* and *B*. *Part A* should be performed individually. *Part B* should be performed in pairs or small groups. The teacher should allow for the majority of Session 1 to be devoted to group work.]

## College Tuition

### Your Assignment:

Based on your research during the last few days, you may have realized that the cost of a college education in the United States can be expensive. During this performance task, you will use a spreadsheet and your knowledge of functions and statistics to predict the future cost of college tuition.



**Steps you will be following:**

To accomplish this, you will use a spreadsheet to help perform the following:

1. Gather data on the past year's tuition amounts.
2. Analyze the data and choose a model type that will best predict the future tuition total.
3. Develop a model equation based on the model type chosen.
4. Predict the total tuition amount for a 2-year or a 4-year college education in the near future.
5. Compare the predicted total tuition amount using the model equation with the total predicted tuition amount you calculated prior to the start of this task.
6. Compare tuition amounts at the college you chose with the average tuition amounts of all public 4-year colleges in the United States.
7. Predict the total tuition amount at a 2-year or a 4-year college education in the distant future.

**Part A****Past Year's Tuition**

During the past few days, each of you chose a specific college or university to research. In order to predict the total tuition amount at that college or university, you must first research past year's tuition amounts for that school.

You will use a computer to search for these data specific to the school you chose. Your search will provide you with the combined cost for tuition and school-related fees at your school over the past several years. Gather these data and enter them into a spreadsheet. The data must include the tuition and school-related fees, as one total dollar amount, for the past 10 years.

[Note: With the data provided on the simulated searches in this example, that will be for the years 2001–2010.]

**Part B****Choosing a Model**

After you have collected your data and entered it into the spreadsheet, get into pairs or groups of 3 or 4. In your group, you will analyze each team member's data by determining the following:

- what the data look like graphically
- what outliers, if any, exist
- what model type, either linear or nonlinear, best fits the data

As a group, decide which model type will be used to determine the function (model equation) that will predict each member's future tuition amounts at his/her chosen school. The model types may or may not be the same for all group members.

**Model Types**

Member 1: \_\_\_\_\_

Member 2: \_\_\_\_\_

Member 3: \_\_\_\_\_

Member 4: \_\_\_\_\_

Look for similarities and differences in each group member's data. Discuss some reasons why the data cause the model types for each group member to be the same or different.

[Note: Allow 5-10 minutes at the end of Session 1 for a whole class discussion about what was discovered during the group work. Students should discuss the reasons they came up with for why the model types in their individual groups, and the class as a whole, may or may not be the same.]

**End of Session 1**

## Session 2

*[Assumptions: All work involving the use of spreadsheet is based on the assumption that students know how to enter data, how to use the chart wizard (or other graphing utility) to create graphs, and how to enter formulas—particularly for linear and exponential regressions, mean, and standard deviation.]*

*[Note: Session 2 of the performance task consists of three parts: C, D, and E. All parts should be performed individually. Part C will make use of the prework involving student's predicted tuition amounts based on the "Tuition and fees rise more than 8% at U.S. public colleges" news article they read and the current year's tuition amount for their chosen school. Part E will require students to analyze their work based on information they read in the news articles "Tuition Hikes of the Downturn" and "The State of the Union on college costs."]*

### Part C

#### Predicting Future Tuition

*[Note: Have students take out the predicted tuition amounts they calculated as part of their prework assignment. Ask the students to summarize what they learned from the "Tuition and fees rise more than 8% at U.S. public colleges" news article they read as part of the prework and how the article guided them in calculating their predicted tuition amounts.]*

When you did your previous group work for this performance task, you each determined the model type that fit your data best. Now, use your spreadsheet to determine the model equation that will be used to predict the tuition amount, as a single dollar amount, at the school you chose.

*[Note: Using actual 4-digit years (2001, 2002, 2003, etc.) as opposed to whole-number years (1, 2, 3, etc.) will result in very different model equations. The equation using the actual 4-digit years will not give tuition amounts appropriate to this problem. Teachers should instruct students to either use whole-number years or try both types of year inputs and make their own judgments as to the appropriate model equation to use.]*

1. Write your model equation here. Show or explain how you found your answer.

2. Use your spreadsheet and your model equation to predict the total 2-year or 4-year tuition amount for the school you chose. Your prediction should begin with the school year that you are first eligible to attend college. Write the tuition amounts below, as well as the total for all years. Be sure to include each year.

Next, compare these total predicted tuition amounts:

- the prediction based on your model equation and spreadsheet data from above
  - the prediction you made after reading the “Tuition and fees rise more than 8% at U.S. public colleges” news article prior to starting this task, using the current year’s tuition amount from your particular school
3. Are these predicted amounts similar or different? Explain why these amounts are similar or different. What does this suggest about the rate of increase for both predicted calculations?

**Part D****Comparing Individual Tuition  
and Average Tuition**

The following table shows actual data for the average tuition costs for all 4-year public universities in the United States.

**Annual Tuition Average  
of Public 4-Year Colleges**

| <b>School Year</b> | <b>Yearly Amount (\$)</b> |
|--------------------|---------------------------|
| 2000-2001          | 3501                      |
| 2001-2002          | 3735                      |
| 2002-2003          | 4046                      |
| 2003-2004          | 4587                      |
| 2004-2005          | 5027                      |
| 2005-2006          | 5351                      |
| 2006-2007          | 5666                      |
| 2007-2008          | 5943                      |
| 2008-2009          | 6312                      |
| 2009-2010          | 6695                      |

Use your spreadsheet to compare the data you found on your college's tuition amounts over a 10-year period with the data for the average tuition amounts at 4-year public colleges.

Suppose the data for the average tuition amounts for all 4-year public colleges were used to create a new model equation. Also, suppose this model equation was used to predict the future college tuition amounts.

4. Which model equation would have a higher correlation factor:

- the model equation created from the average tuition amounts; or
- the model equation created from the tuition amounts at your chosen school?

Explain why that model equation has a higher correlation factor. What does this suggest about how reliable each model is for predicting future college tuition amounts? Explain your reasoning. You may use a combination of diagrams, mathematical expressions/equations, and words.

### ***Part E***

#### **Predicting Tuition for the Next Generation**

In this last part of the task, you will predict the total tuition amount someone in a future generation will be expected to pay.

Use your spreadsheet and the model equation you determined for your college to predict the total tuition amount for a family that has a child born 10 years from now. You should assume that this child will:

- attend the college you chose,
  - begin college at age 18, and
  - attend for the full length of time (either 2 or 4 years).
5. Show how you determined the predicted total college tuition amount for this person. You may use a combination of diagrams, mathematical expressions/equations, and words.

Consider the two articles, "Tuition Hikes of the Downturn" and "The State of the Union on college costs," you read prior to starting this task.

6. Use these articles to help justify why predicting college tuition costs too far into the future, beyond a few years, might not be reliable. Cite information from each article that supports your reasoning.

[Note: Students should have both articles available during this portion of the task.]

**End of Session 2**

*Sample Top-Score Response:*

**Prework** (Not Scored)

Should include:

Name of school, type of school, and current full-year tuition amount

E.g.,

UMASS Boston, 4-year University, \$11,407 for 2011-2012 in-state tuition and fees

Source: [http://www.umb.edu/bursar/tuition\\_and\\_fees/](http://www.umb.edu/bursar/tuition_and_fees/)

The predicted cost for college tuition should apply a yearly increase close to 8.3% for each year's tuition amount.

E.g.,

Start college for school year 2013-2014

First year's predicted tuition amount:  $\$11,407 \times 1.083 \times 1.083 = \$13,379$

The sum of 2 or 4 consecutive years' projected tuition amounts should be made, approximately 1-3 years from the current year.

E.g.,

First year predicted amount: \$13,379

Second year:  $\$13,379 \times 1.083 = \$14,489$

Third year:  $\$14,489 \times 1.083 = \$15,692$

Fourth year:  $\$15,692 \times 1.083 = \$16,994$

Total =  $\$13,379 + \$14,489 + \$15,692 + \$16,994 = \mathbf{\$60,554}$

**Part A** (Not Scored)

The spreadsheet should show data in two columns. The first column should include the most recent 10 years shown on the tuition data site. The second column should include the corresponding yearly tuition amounts for the college chosen by the student. For example:

|    | <b>Year</b> | <b>UMASS Boston<br/>In-state Tuition</b> |
|----|-------------|--|
| 1  | 2001-2002   | 5170                                     |
| 2  | 2002-2003   | 6229                                     |
| 3  | 2003-2004   | 8142                                     |
| 4  | 2004-2005   | 9112                                     |
| 5  | 2005-2006   | 9028                                     |
| 6  | 2006-2007   | 9143                                     |
| 7  | 2007-2008   | 9060                                     |
| 8  | 2008-2009   | 9353                                     |
| 9  | 2009-2010   | 10,611                                   |
| 10 | 2010-2011   | 10,611                                   |

**Part B** (Not Scored)

Model types (linear or nonlinear – exponential or quadratic) should be listed for each group



member.

Some notes or reflections should portray some similarities and some differences among the data in the group. The students should provide reasonable explanations why the data have these similarities and differences.

**Part C** (Scored)

1. The student determines the correct linear (or nonlinear) model that best fits their specific data. The model is written either as an equation or as a function. The student should show the formula and data they used in the spreadsheet in order to determine their model equation. The student is able to interpret the output from the formula correctly in order to write the model equation.

E.g., using data from part A

Formula = "=LINEST(C2:C11,A2:A11)"

Output = 519.133, 5790.667

Model equation:  $f(x) = 519.133x + 5790.667$

2. The student correctly applies the model equation they determined for years in the near future. The student substitutes the appropriate 2 or 4 school years for  $x$  into the model equation and gets the predicted college tuition amount,  $y$ . The student adds the amounts for each year's output,  $y$ , to determine the total predicted college tuition amount. For example:

| Year                |           | UMASS Boston<br>In-state Tuition |
|---------------------|-----------|----------------------------------|
| 1                   | 2001-2002 | 5170                             |
| 2                   | 2002-2003 | 6229                             |
| 3                   | 2003-2004 | 8142                             |
| 4                   | 2004-2005 | 9112                             |
| 5                   | 2005-2006 | 9028                             |
| 6                   | 2006-2007 | 9143                             |
| 7                   | 2007-2008 | 9060                             |
| 8                   | 2008-2009 | 9353                             |
| 9                   | 2009-2010 | 10,611                           |
| 10                  | 2010-2011 | 10,611                           |
| <b>slope</b>        |           | <b>y-intercept</b>               |
| <b>Linear model</b> | 519.133   | 5790.667                         |
| 11                  | 2011-2012 | \$11,501                         |
| 12                  | 2012-2013 | \$12,020                         |
| 13                  | 2013-2014 | \$12,539                         |
| 14                  | 2015-2016 | \$13,059                         |
| 15                  | 2016-2017 | \$13,578                         |
| 16                  | 2017-2018 | \$14,097                         |
| <b>Total</b>        |           | <b>\$53,273</b>                  |

3. The student notices a similarity or difference between the two predicted total amounts and is able to reasonably explain why the similarity or difference exists. If the predicted total using the model equation is less than the predicted total based on the article, the student explains that the average yearly increase must be less than 8.3%. If the predicted total using the model equation is more than the predicted total based on the article, the student relates that the average yearly increase must be greater than 8.3%.

For example:

Total predicted tuition based on model equation: \$53,272

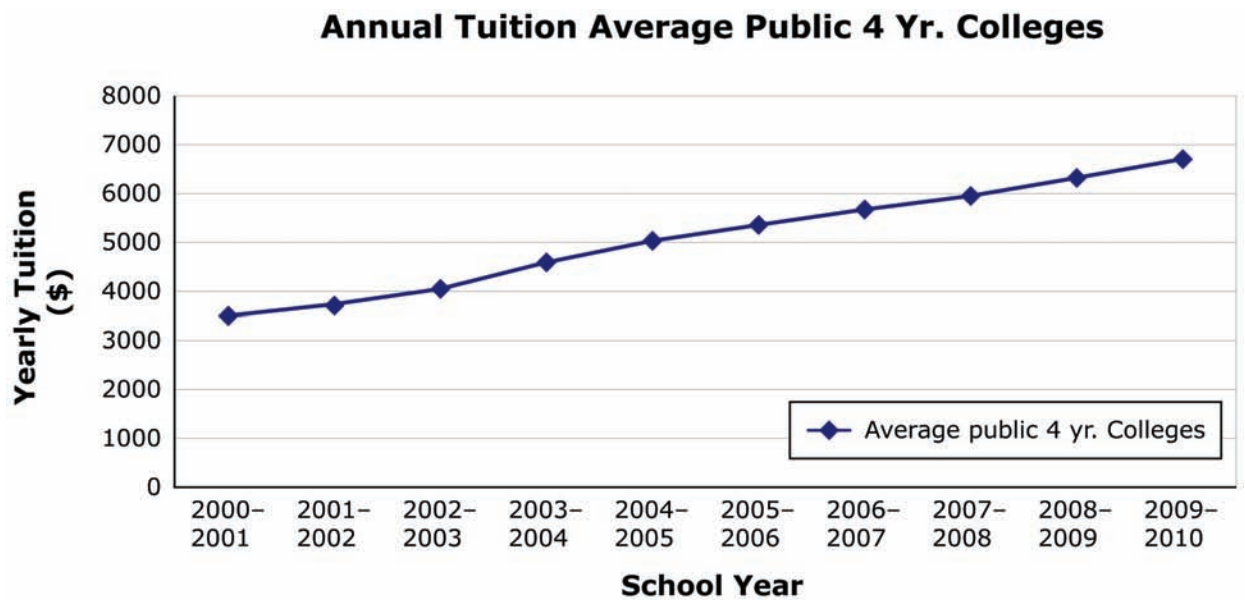
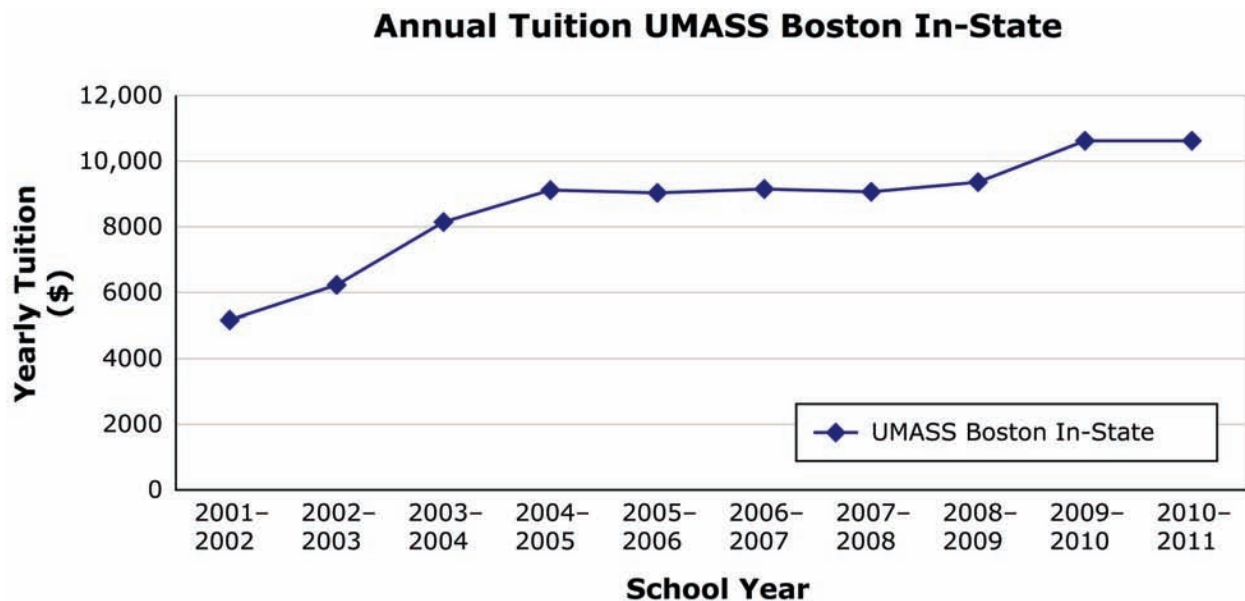
Total predicted tuition based on current year's tuition and data from article: \$60,554

These predicted amounts differ by a significant amount, \$7282. This is most likely because the rate at which the model predicts tuition to increase each year is less than the 8.3% average mentioned in the article.

**Part D** (Scored)

4. The student gives a reasonable explanation of why the average tuition model equation (most likely) has the higher correlation factor. The student relates the almost perfectly linear relationship of the average data to having a line of best fit that produces a model equation very close to the actual data points, creating a high correlation factor. The data for the chosen school, however (most likely), do not have as close a linear relationship, so the

line that best fits that data will have a (slightly) lower correlation factor. For example:



The student offers a reasonable explanation as to how the reliability of each model and the correlation factor associated with each model are related. The student recognizes that the higher the correlation factor, the more reliable the model is expected to be.

For example:

The model equation for the average tuition data is  $f(x) = 362.612x + 3091.933$ . When I input the years 1-10 into this equation and correlate these outputs to the actual averages, I get a correlation factor of 0.99739. This is a very high correlation factor, which is expected since the data on the graph show an almost linear relationship with the given data. This

high correlation factor indicates that the model for the average tuition data is very reliable.

When I input the years 1-10 into the model equation I found for my college and correlate these outputs to the actual averages, I get a correlation factor of 0.90441. This makes sense that the correlation factor is somewhat lower because the data for UMASS' tuition is not quite as linear as the data for the average tuition. While this is not as high a correlation factor as the average tuitions' model, it still is fairly high and indicates that the model equation to predict the tuition for my college is still fairly reliable.

**Part E** (Scored)

5. The student correctly applies the model equation from part C. The student recognizes that the starting value of  $x$  must be 28 ( $10 + 18$ ) more than the value of  $x$  for the current year. The student substitutes the appropriate 2 or 4 school years for  $x$  into the model equation and gets the predicted college tuition amount,  $y$ . The student adds the amounts for each year's output,  $y$ , to determine the total predicted college tuition amount for someone in the next generation. For example:

**28 years from now**

|              |           |                  |
|--------------|-----------|------------------|
| 40           | 2040–2041 | \$26,556         |
| 41           | 2041–2042 | \$27,075         |
| 42           | 2042–2043 | \$27,594         |
| 43           | 2043–2044 | \$28,113         |
| <b>Total</b> |           | <b>\$109,338</b> |

I determined that this person would be attending college 28 years from now, beginning in the year 2040. I used the 4-year period beginning in that year to determine the total tuition using the model equation for UMASS found in part C.

6. The student gives a reasonable explanation to justify why predicting college tuition costs too far into the future might not be reliable. The student cites details relating to how the varying percentage amount increases by decade shown in the Annual Average Tuition Increases (Inflation-Adjusted) by Sector table [in article 2] indicates that a single percent increase, as given in the model equation, cannot be relied on. The student relates the impact that the economy plays in driving tuition rates up [in articles 2 and 3], and that without a clear prediction of the future of the economy, it is hard to predict how college tuition rates will increase. The student discusses that differences exist in average tuition increases based on the type of school it is (public vs. private). The factors that influence these different percent increases are discussed along with the possibility that this trend may not always be the case [in article 2]. The student cites concerns outlined in President Obama's State of the Union address [in article 3] that government may need to take some control in the future by limiting the amount of government aid colleges receive if tuition increases continue climbing at their current rate. The student relates this to the possibility that tuition increases may start to decline, thus making their model unreliable several years into the future.

**Scoring Notes:**

Each question in the scored parts is evaluated individually. The total number of points is determined by adding the points assigned for each question.

**Scoring Rubric:**

Scoring Rubric for **Part C**:

**Part C**, Question 1: Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student shows a thorough understanding of how to use a spreadsheet to determine a regression function and interpret the result of the output. The student uses the correct regression formula in the spreadsheet, references the correct data values for the function, and interprets each output number from the formula to write the correct model equation (e.g., A linear regression formula outputs two numbers. The first number represents the slope of the function and the second number represents the  $y$ -intercept).

**1 point:** The student shows partial understanding of how to use a spreadsheet to determine a regression function and interpret the result of the output. The student uses the correct regression formula in the spreadsheet, references the correct data values for the function, but misinterprets the output numbers from the formula. **OR** The student uses the correct regression formula in the spreadsheet, but references the incorrect data values for the function. However, the student is able to interpret the output numbers from the formula correctly.

**0 points:** The student shows inconsistent understanding of how to use a spreadsheet to determine a regression function and interpret the result of the output. The student does not use the correct regression formula in the spreadsheet, and does not interpret the output numbers from the formula correctly to write a model equation.

**Part C**, Question 2: Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student shows a thorough understanding of how to use a spreadsheet to apply a regression function to find a total predicted tuition amount. The student projects out the starting year and ending year correctly for when they plan to attend the college. The student uses those years in the model equation to find the predicted tuition amounts for those years and then sums the 2- or 4-year amounts for one total.

**1 point:** The student shows partial understanding of how to use a spreadsheet to apply a regression function to find a total predicted tuition amount. The student projects out the starting year and ending year correctly for when they plan to attend the college. The student uses those years in the model equation to find the predicted tuition amounts for those years but does not find their total sum. **OR** The student projects out the incorrect starting year and ending year but applies the model equation correctly to find predicted tuition amounts for those years. The student sums the amounts for one total.

**0 points:** The student shows inconsistent understanding of how to use a spreadsheet to

apply a regression function to find a total predicted tuition amount. The student may or may not project out the starting year and ending year correctly. However, the student applies the model equation incorrectly and does not find or incorrectly finds the sum the 2- or 4-year amounts.

**Part C**, Question 3: Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student shows a thorough understanding of how to compare predicted tuition amounts using more than one method for predicting. The student correctly identifies whether the two predicted tuition sums are similar or different. The student relates the rate of increase mentioned in the article to the rate of increase used in the model equation.

**1 point:** The student shows partial understanding of how to compare predicted tuition amounts using more than one method for predicting. The student correctly identifies whether the two predicted tuition sums are similar or different but does not relate the rate of increase mentioned in the article to the rate of increase used in the model equation.

**0 points:** The student shows inconsistent understanding of how to compare predicted tuition amounts using more than one method for predicting. The student incorrectly identifies whether the two predicted tuition sums are similar or different.

Scoring Rubric for **Part D**:

**Part D**, Question 4: Responses to this item will receive 0-3 points, based on the following:

**3 points:** The student shows a thorough understanding of how to analyze data in terms of correlation and uses that knowledge to make judgments about the reliability of models. The student identifies which model has a higher correlation factor and provides an accurate explanation as to why it is higher. The student reasonably relates the correlation factor to the reliability of each model. The student provides a complete and accurate explanation for all aspects of this part using diagrams, expressions/equations, and/or words.

**2 points:** The student shows some understanding of how to analyze data in terms of correlation and uses that knowledge to make judgments about the reliability of models. The student identifies which model has a higher correlation factor but does not provide a complete explanation as to why it is higher. The student reasonably relates the correlation factor to the reliability of each model. The student provides a complete and accurate explanation for most aspects of this part using diagrams, expressions/equations, and/or words.

**1 point:** The student shows partial understanding of how to analyze data in terms of correlation and uses that knowledge to make judgments about the reliability of models. The student identifies which model has a higher correlation factor and provides an accurate explanation as to why it is higher. The student incompletely or inaccurately relates the correlation factor to the reliability of each model. The student provides a complete and accurate explanation for some aspects of this part using diagrams, expressions/equations, and/or words.

**0 points:** The student shows inconsistent understanding of how to analyze data in terms of correlation and uses that knowledge to make judgments about the reliability of models. The

student may or may not identify which model has a higher correlation factor but provides no explanation or an incorrect explanation as to why it is higher. The student incompletely or inaccurately relates the correlation factor to the reliability of each model. The student provides no complete or accurate explanation for any aspect of this part using diagrams, expressions/equations, and/or words.

Scoring Rubric for **Part E**:

**Part E**, Question 5: Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student shows a thorough understanding of how to apply the model equation to find a total predicted tuition amount in the distant future. The student projects out the starting year and ending year correctly. The student uses the appropriate years in the model equation to find the predicted sum of the tuition for a student attending college in the future.

**1 point:** The student shows partial understanding of how to apply the model equation to find a total predicted tuition amount in the distant future. The student projects out the starting year and ending year correctly but applies the model equation incorrectly.

**0 points:** The student shows inconsistent understanding of how to apply the model equation to find a total predicted tuition amount in the distant future. The student does not project out the starting year and ending year correctly. The student does not apply the model equation correctly to find the predicted sum of the tuition for a student attending college in the future.

**Part E**, Question 6: Responses to this item will receive 0-3 points, based on the following:

**3 points:** The student shows a thorough understanding of how to interpret information regarding the future of college tuition rate increases presented in news articles and justifies conclusions based on the analysis. The student gives a complete and reasonable explanation as to why predicting college tuition costs too far into the future might not be reliable. The student supports his or her reasoning with at least 3 statements coming from both related articles.

**2 points:** The student shows some understanding of how to interpret information regarding the future of college tuition rate increases presented in news articles and justifies conclusions based on the analysis. The student gives a reasonable explanation as to why predicting college tuition costs too far into the future might not be reliable. The student supports his or her reasoning with at least 2 statements coming from both related articles.

**1 point:** The student shows partial understanding of how to interpret information regarding the future of college tuition rate increases presented in news articles and justifies conclusions based on the analysis. The student gives an incomplete or partially correct explanation as to why predicting college tuition costs too far into the future might not be reliable. The student supports his or her reasoning with at least 2 statements coming from both related articles. Or the student supports his or her reasoning with statements coming from only one article.

**0 points:** The student shows inconsistent understanding of how to interpret information regarding the future of college tuition rate increases presented in news articles and justifies conclusions based on the analysis. The student gives an incorrect explanation as to why predicting college tuition costs too far into the future might not be reliable. The student may or may not support his or her reasoning with statements from both related articles.

## Article #1

<http://www.usatoday.com/news/education/story/2011-10-25/public-college-costs-increase/50919598/1>

### Tuition and fees rise more than 8% at U.S. public colleges

By Mary Beth Marklein, USA TODAY

Updated 10/26/2011 2:12 AM

Tuition and fees at America's public colleges rose more than 8% this year as a weakened economy and severe cuts in state funding took their toll, a report out today says.



By Jacquelyn Martin, AP

Gan Golan of Los Angeles, dressed as the "Master of Degrees," holds a ball and chain representing his college loan debt during an "Occupy D.C." protest Oct. 6.

Public four-year universities charged residents an average of \$8,244, up 8.3% from last year, while public two-year schools charged an average of \$2,963, up 8.7%, says the report by the non-profit [College Board](#). About 80% of the nation's undergraduates attend public institutions.

That increase is more than double the inflation rate of 3.6% between July 2010 and July 2011. Family earnings dropped across all income levels. And state funding per student declined by 4% in 2010, the latest year available, and 23% over the past decade, the report says.



## HS Mathematics Sample PT Form Claim 4

### ***College costs***

Average estimated costs\* for full-time undergraduates in 2011-12, before grant aid\*\*:

Public two-year commuter student: \$15,286

Public four-year, in-state, lives on campus: \$21,447

Public four-year, out-of-state, lives on campus: \$33,973

Private non-profit four-year, lives on campus: \$42,224

\*Costs include tuition and fees, room and board, books and supplies, transportation and other expenses.

\*\*About one-third of full-time undergraduates receive no grant aid.

*Source: College Board*

Molly Corbett Broad, president of the [American Council on Education](#), called the findings "sadly familiar," and said the drop in state support was particularly troubling. "It has become all too common for state legislatures to dip into the pockets of students and families to balance state budgets," she says.

The tuition and fee hike is not the worst of the decade — that occurred in 2004, when sticker prices rose 11% beyond inflation from the previous year.

The report says there may be some good news: a rise in federal student aid — including tax credits and deductions — is blunting the impact for most families. "At a time when students and families are ill-equipped to manage additional expenses, student financial aid is more important than ever," report author Sandy Baum says.

Net price — the published price minus grants and tax breaks — at public four-year colleges averaged \$2,490, the report found.

About two-thirds of undergraduates receive grant aid, which averaged \$6,539 last year. Average federal loans averaged \$4,907. Borrowing by students and parents increased about 2% from 2009-10 to 2010-11.

Borrowing from private sources declined for the third straight year. In Denver today, [President Obama](#) will announce a plan through which students can consolidate their debt and reduce their interest rates. The plan also will allow borrowers to cap their student loan payments at 10% of discretionary income.

## Article #2

<http://www.insidehighered.com/news/2010/10/28/tuition#ixzz1mbLlzzuN>

Inside Higher Ed

### Tuition Hikes of the Downturn

October 28, 2010 - 3:00am

By [Scott Jaschik](#)

Tuition is up (no surprise) and this year the percentage increases for public and private four-year colleges and universities are higher than they were last year. Generally, the percentage increases at public institutions are larger than those at privates (which are more expensive to start with). Those trends are standard for tight economic times, when states cut budgets and try to make up for shortfalls with larger tuition increases, and when many private colleges worry that sticker shock will scare away families and so tend to moderate price increases.

Across the board, the increases exceed the inflation rate of about 1.2 percent for the last year, which, while low, was higher than the slightly negative rate of the year before.

Those are the key findings from this year's [annual survey on college prices](#) (and a [companion survey on student aid](#)) being released today by the College Board. In many respects, the data extend trends that were evident last year as well. Here are the overall figures for the 2010-11 academic year:

**Tuition and Fees by Sector**

| Sector  | 2010-11 Tuition and Fees | One-Year Dollar Increase | One-Year % Increase | Previous Year's % Increase |
|---|--------------------------|--------------------------|---------------------|----------------------------|
| Private, nonprofit four-year colleges             | \$27,293                 | \$1,164                  | 4.5%                | 4.4%                       |
| Public four-year colleges, in-state residents     | \$7,605                  | \$555                    | 7.9%                | 6.5%                       |
| Public four-year colleges, out-of-state residents | \$19,595                 | \$1,111                  | 6.0%                | 6.2%                       |
| Community colleges                                | \$2,713                  | \$155                    | 6.0%                | 7.3%                       |
| For-profit colleges                               | \$13,935                 | \$679                    | 5.1%                | 6.5%                       |

For room and board, public increases also outpaced the privates, and privates are also more expensive. The average public college rate is going up by 4.6 percent, to \$8,535, and the average private rate is going up by 3.9 percent, to \$9,700. Those figures are for four-year institutions only, as the pool of community colleges and for-profit colleges charging for room and board remains small.

As is the case every year, College Board officials stress that the data show that most colleges -- however much their prices frustrate students and families -- are not in the mid-

## HS Mathematics Sample PT Form Claim 4

\$50,000 range that attracts so much attention. Total expenses for a private four-year institution are, on average, just under \$37,000 a year. But because the most famous private institutions tend to be well above that average, many people assume tuition rates are even higher than they are. (At Harvard University, an undergraduate's total costs this year, typical for those at elite private research universities and liberal arts colleges, are estimated by the university to be between \$53,950 and \$56,750.)

Many of the data in the report focus on the impact of state budget shortfalls on public colleges. For instance, in comparing inflation-adjusted average tuition increases from the last three decades, the College Board finds that over that time, the rate of increase has dropped for private four-year institutions and gone up for public four-year institutions. Further, while the rate of increase at private institutions was greater than that of publics in the 1980s, it is now smaller.

### Annual Average Tuition Increases (Inflation-Adjusted) by Sector

| Sector             | 1980-1<br>to<br>1990-1 | 1990-1<br>to<br>2000-1 | 2000-1<br>to<br>2010-1 |
|--------------------|------------------------|------------------------|------------------------|
| Private four-year  | 5.1%                   | 2.6%                   | 3.0%                   |
| Public four-year   | 4.2%                   | 3.3%                   | 5.6%                   |
| Community colleges | 3.9%                   | 3.2%                   | 2.7%                   |

The College Board's report on student aid notes that the past two years -- which have seen significant increases in tuition at many public colleges and universities and growing economic pressures on many families -- have seen a rapid expansion in aid packages.

From 2008-9 to 2009-10, grant aid per full-time equivalent undergraduate increased by about 22 percent (or \$1,073) and federal loans increased by 9 percent (about \$408). Particularly notable, the College Board report said, was the increase in the maximum Pell Grant of 16 percent in constant dollars in 2009-10, the largest one-year increase in program history. The total Pell budget reached \$28.2 billion, divided among 7.7 million students.

Sandy Baum, a policy analyst for the College Board and co-author of the reports being issued, said that the tuition figures "were not very surprising," given the state of the economy. "I don't think anybody thought public tuition would go up only 2 percent this year."

She urged educators and policy-makers to pay more attention to the long-term issues raised by this year's data. She noted, for example, that the impact of tuition increases on low-income students has been mitigated in part by the strong support for the growth in Pell Grants -- growth that probably will not be matched in the years ahead. "No matter what kind of Congress we get, the idea that Pell Grants will keep growing at this rate is unlikely," she said.

Baum said that in many ways she sees the tuition trends posing more of a threat ahead to public higher education than to private colleges. She said that some private institutions -- those that are being forced to give so much aid to attract students that they can't balance

## HS Mathematics Sample PT Form Claim 4

their books -- are in danger. But she said that the basic financial model for most privates, in which some students pay enough to subsidize others, is sound.

For public higher education, however, she said she feared that "the basic model may no longer be sustainable." While states are likely to restore some support for higher education as the economy improves, she said, it seems unlikely that enough support will be provided to maintain tuition at affordable levels. She said she anticipates public colleges having to consider more radical changes in how they provide education, ideally using means that cut costs. She noted that while technology has to date not cut costs in providing higher education, that may not be the case in the future.

If new models fail to provide more students with quality education, she said, "we could lose public higher education, and that would be a huge social failure."

### Article #3

<http://www.changinghighereducation.com/2012/01/so-let-me-put-colleges-and-universities-on-notice-if-you-cant-stop-tuition-from-going-up-the-funding-you-get-from-taxpay.html>

### The State of the Union on college costs

*So let me put colleges and universities on notice: If you can't stop tuition from going up, the funding you get from taxpayers will go down. Higher education can't be a luxury. It is an economic imperative that every family in America should be able to afford.*

Barak Obama, [State of Union 2012](#)

Does this speech signal that the time has finally arrived when the government - which pays a good part of the bill - will step in to limit the rapid and seemingly never ending growth of tuition? In normal times, the answer would likely be "yes" given that politicians from [both sides of the aisle](#) have been introducing bills that would cap tuition in one way or another for almost a decade. Thus, we might expect to see a quick moving bipartisan effort.

These, of course, are not times when bipartisan efforts go very far, so Obama's statements will probably push Republicans into fierce opposition to the idea. The [response](#) of [Representative Virginia Foxx](#), the North Carolina Republican who is chairwoman of the House Higher Education subcommittee, is probably a pretty good representation of what we will now hear from the Republican side:

*The president is saying that people can't afford to go to college anymore, and that just simply is not true. Tuition is too high at most schools, but it isn't the job of the federal government to punish those schools. It's very arbitrary, and the president sounds like a dictator.*

## HS Mathematics Sample PT Form Claim 4

So this probably won't be the tipping point for this issue. But before the higher education community breathes a sigh of relief, its members should note that a President of the United States views the issue as important enough, with enough broad voter appeal, to put it into a State of the Union address, and he is continuing to speak about it at [public events](#). It would be surprising if we didn't hear a lot more over the next two years about the relationship between tuition increases and taxpayer support. And, despite the negative initial overall response of Representative Foxx, it should be noted that she agreed that tuition is too high at most schools - hardly the position that makes a strong ally in this matter.

The reported responses from the academic community to Obama's speech, sadly, fall pretty much as one would anticipate -The current system is close to perfect, and any constraints (fiscal or administrative) will lead to declines in educational outcomes. This is indeed the likely outcome if educational institutions try to handle the constraints without changing their basic approach.

However this speech makes it increasingly clear that the reality must be faced - it is simply not possible for higher education costs to increase at 3% above inflation forever, and the end of the period of rapid increases is getting closer. Educational leaders that refuse to come to grips with this reality are ensuring that the negative outcomes they describe will indeed occur.

It is highly likely that the changes that will be required will involve things that most people in traditional higher education find undesirable because they break with comfortable traditional standards of "how things should be done". But the economic realities of the United States (and most of the rest of the world) are such that "undesirable" actions have been, are, and will be required of almost every segment in order to transition to new, viable configurations. Does higher education have the leadership to rise to the challenge of this kind of transformative change, or will it simply sink into mediocrity while defending the *status quo*?

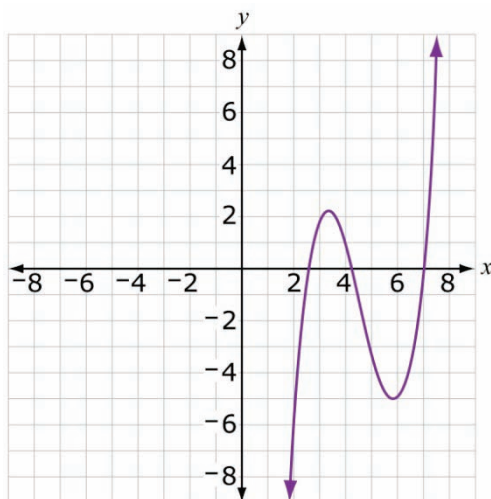
MAT.HS.SR.1.00FIF.K.082

|  |   |
|--|---|
| Sample Item ID:  | MAT.HS.SR.1.00FIF.K.082   |
| Grade:   | HS  |
| Claim(s):  | <b>Claim 1: Concepts and Procedures</b><br>Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency. |
| Assessment Target(s):                                    | 1 K: Understand the concept of a function and use function notation.  |
| Content Domain:  | Functions   |
| Standard(s):   | F-IF.1  |
| Mathematical Practice(s):                                | 4, 6  |
| DOK:   | 2   |
| Item Type:   | SR  |
| Score Points:  | 1   |
| Difficulty:  | M   |
| Key:   | YNYN  |
| Stimulus/Source:   |   |
| Target-specific attributes (e.g., accessibility issues): |   |
| Notes:   | Multi-Answer Item   |

For numbers 1a – 1d, determine whether each relation is a function.

1a.  $\{(0, 1), (1, 2), (3, 1), (4, 2)\}$   Yes  No

1b.  $y = \pm\sqrt{4 - x^2}$   Yes  No



1c.  Yes  No

1d.  $\{(5, 3), (2, 4), (5, 2)\}$   Yes  No

*Key and Distractor Analysis:*

1a. Y  $\{(0, 1), (1, 2), (3, 1), (4, 2)\}$

All  $x$ -coordinates are unique, so it meets the definition of a function.

1b. N  $y = \pm\sqrt{4 - x^2}$

An input of  $x = 1$  has two corresponding outputs,  $y = \sqrt{3}$  and  $y = -\sqrt{3}$ , so it fails to meet the definition of a function.

1c. Y This is a function since for each value chosen along the  $x$ -axis, there is exactly one  $y$ -value on the graph that corresponds to it.

1d. N This is not a function since the input of 5 has two corresponding output values, 3 and 2.

**MAT.HS.SR.1.00GCO.O.244**

|  |   |
|--|---|
| Sample Item ID:  | MAT.HS.SR.1.00GCO.O.244   |
| Grade:   | HS  |
| Claim(s):  | <b>Claim 1: Concepts and Procedures</b><br>Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency. |
| Assessment Target(s):                                    | 1 O: Prove geometric theorems.  |
| Content Domain:  | Geometry  |
| Standard(s):   | G-CO.9  |
| Mathematical Practice(s):                                | 3, 5  |
| DOK:   | 2   |
| Item Type:   | SR  |
| Score Points:  | 1   |
| Difficulty:  | L   |
| Key:   | D   |
| Stimulus/Source:   |   |
| Target-specific attributes (e.g., accessibility issues): | Visually impaired students may have difficulty accessing this item due to the animation.  |
| Notes:   | Animation item-requires QuickTime video   |

Click on the play button to view an animation of the construction of a parallel line. The animation begins by showing a line and a point above the line.



Parallel\_Line.mov

The steps in the construction result in a line through the given point that is parallel to the given line.

Which statement justifies why the constructed line is parallel to the given line?

- A. When two lines are each perpendicular to a third line, the lines are parallel.
- B. When two lines are each parallel to a third line, the lines are parallel.
- C. When two lines are intersected by a transversal and alternate interior angles are congruent, the lines are parallel.

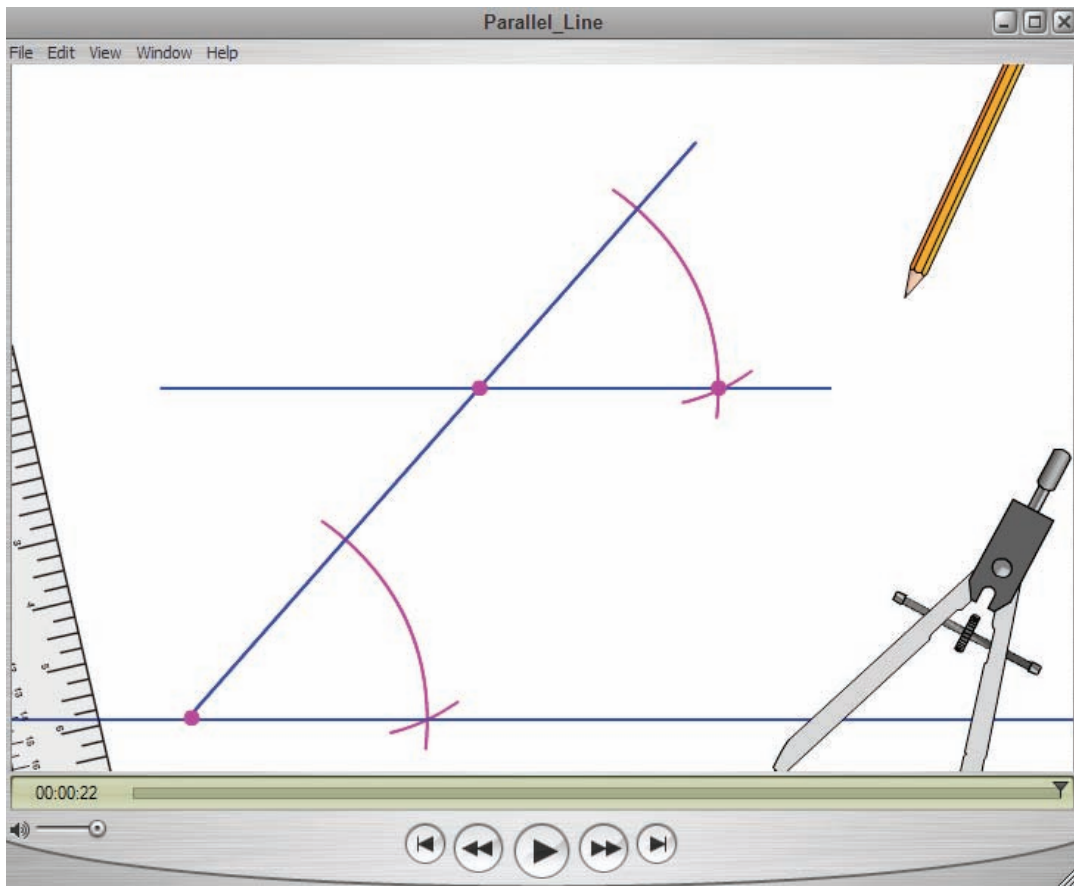


D. When two lines are intersected by a transversal and corresponding angles are congruent, the lines are parallel.

*Key and Distractor Analysis:*

The key is D.

Rationale for choosing correct option: The steps in the construction make a copy of the angle formed between the transversal and the given line, so these angles are congruent. The construction steps have produced two lines intersected by a transversal with a pair of corresponding angles congruent. Option D states a theorem that can be used to conclude that the corresponding angles must be congruent.



Rationale for choosing incorrect options:  
 These are all principles that can be used to prove that two lines are parallel, though only option D applies to this construction.

Mathematics Sample SR Item C1 TA

MAT.HS.SR.1.00NRN.A.152

|  |   |
|--|---|
| Sample Item ID:  | MAT.HS.SR.1.00NRN.A.152   |
| Grade:   | HS  |
| Claim(s):  | <b>Claim 1: Concepts and Procedures</b><br>Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency. |
| Assessment Target(s):                                    | 1 A: Extend the properties of exponents to rational exponents.  |
| Content Domain:  | Number and Quantity   |
| Standard(s):   | N-RN.2  |
| Mathematical Practice(s):                                | 1, 2, 6   |
| DOK:   | 2   |
| Item Type:   | SR  |
| Score Points:  | 2   |
| Difficulty:  | M   |
| Key:   | TTFTF   |
| Stimulus/Source:   |   |
| Target-specific attributes (e.g., accessibility issues): |   |
| Notes:   | Calculator tool must be turned off for this item.   |

For items 1a – 1e, determine whether each equation is True or False.

1a.  $\sqrt{32} = 2^{\frac{5}{2}}$        True       False

1b.  $16^{\frac{3}{2}} = 8^2$        True       False

1c.  $4^{\frac{1}{2}} = \sqrt[4]{64}$        True       False

1d.  $2^8 = (\sqrt[3]{16})^6$        True       False

1e.  $(\sqrt{64})^{\frac{1}{3}} = 8^{\frac{1}{6}}$        True       False

*Scoring Rubric for Multi-part Items:*

*Responses to this item will receive 0-2 points, based on the following:*

**2 points:** TTFTF The student has a solid understanding of how to rewrite expressions involving radical and rational exponents to determine equivalent forms.

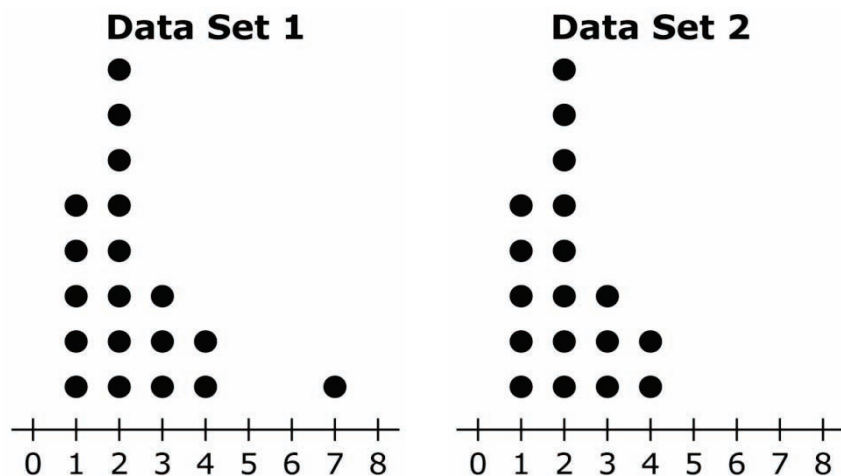
**1 point:** TTFTT, TTTFF, TTTTF, TTFFT, TTTFF The student only has a basic understanding of how to rewrite expressions involving radical and rational exponents. The student can evaluate expressions containing square roots and expressions containing integer exponents as well as some simple rational exponents, such as  $\frac{1}{2}$  or  $\frac{3}{2}$ . The student has difficulty evaluating expressions with cube roots or fourth roots and expressions with roots raised to integer or rational exponents. The student must answer parts a and b correctly, as well as at least one of the remaining parts (exception TTTTT would suggest a guessing pattern).

**0 points:** All other possibilities. The student demonstrates inconsistent understanding of how to rewrite expressions involving radical and rational exponents.

**MAT.HS.SR.1.00SID.P.084**

|  |   |
|--|---|
| Sample Item ID:  | MAT.HS.SR.1.00SID.P.084   |
| Grade:   | HS  |
| Claim(s):  | <b>Claim 1: Concepts and Procedures</b><br>Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency. |
| Assessment Target(s):                                    | 1 P: Summarize, represent and interpret data on a single count or measurement variable.   |
| Content Domain:  | Statistics & Probability  |
| Standard(s):   | S-ID.2  |
| Mathematical Practice(s):                                | 1, 2, 7   |
| DOK:   | 2   |
| Item Type:   | SR  |
| Score Points:  | 2   |
| Difficulty:  | M   |
| Key:   | G1, E, G1   |
| Stimulus/Source:   |   |
| Target-specific attributes (e.g., accessibility issues): |   |
| Notes:   | Only one selection per row can be made.   |

The frequency distributions of two data sets are shown in the dot plots below.



For each of the following statistics, determine whether the value of the statistic is greater for Data Set 1, equal for both data sets, or greater for Data Set 2.

|                    | <b>Greater for Data Set 1</b> | <b>Equal for Both Data Sets</b> | <b>Greater for Data Set 2</b> |
|--------------------|-------------------------------|---------------------------------|-------------------------------|
| Mean               |                               |                                 |                               |
| Median             |                               |                                 |                               |
| Standard Deviation |                               |                                 |                               |

Click on the box that represents your choice for each measure.

*Scoring Rubric:*

**Key:**

- Row 1: Greater for Data Set 1
- Row 2: Equal for both data sets
- Row 3: Greater for Data Set 1

**Scoring Rubric:**

Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student has a thorough understanding of how to apply mathematical concepts and carry out mathematical procedures for comparing the center and spread of two different data sets, where one set contains an outlier. The student correctly indicates how the inclusion of the outlier affects both the measures of center (mean, median) and spread (standard deviation).

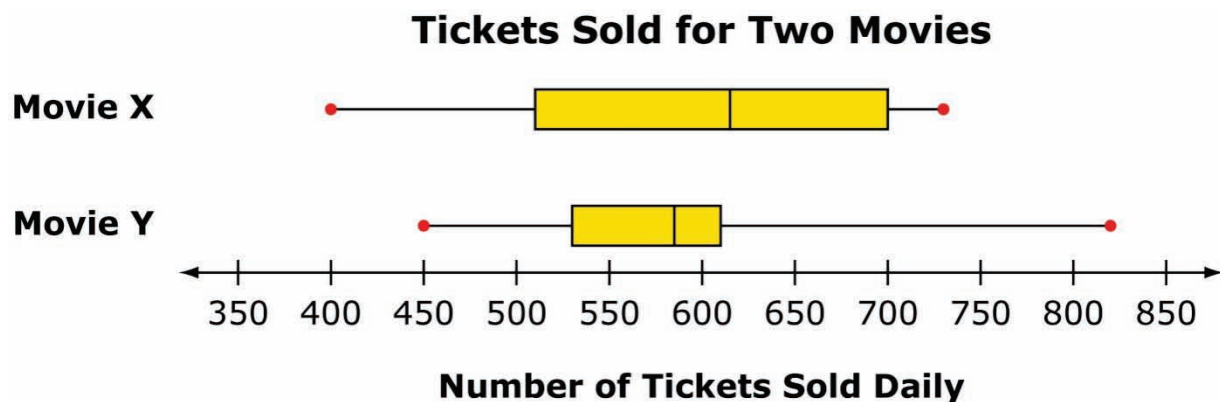
**1 point:** The student has a basic understanding of how to apply mathematical concepts and carry out mathematical procedures for comparing the center and spread of two different data sets, where one set contains an outlier. The student correctly identifies how the outlier affects the mean and median but not the standard deviation. **OR** The student correctly identifies how the outlier affects the standard deviation and the mean or median.

**0 points:** The student has an inconsistent understanding of how to apply mathematical concepts and carry out mathematical procedures for comparing the center and spread of two different data sets, where one set contains an outlier. The student fails to correctly identify how the outlier affects the mean, median, and standard deviation. **OR** The student correctly identifies how the outlier affects the standard deviation but not the mean or median.

**MAT.HS.SR.1.00SID.P.482**

|  |   |
|--|---|
| Sample Item ID:  | MAT.HS.SR.1.00SID.P.482   |
| Grade:   | HS  |
| Claim(s):  | <b>Claim 1: Concepts and Procedures</b><br>Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency. |
| Assessment Target(s):                                    | 1 P: Summarize, represent and interpret data on a single count or measurement variable.   |
| Content Domain:  | Statistics & Probability  |
| Standard(s):   | S-ID.1  |
| Mathematical Practice(s):                                | 1, 4  |
| DOK:   | 2   |
| Item Type:   | SR  |
| Score Points:  | 2   |
| Difficulty:  | L   |
| Key:   | CTT   |
| Stimulus/Source:   |   |
| Target-specific attributes (e.g., accessibility issues): |   |
| Notes:   | Only one selection per row can be made.   |

A movie theater recorded the number of tickets sold for two movies each day during one week. Box plots of the data are shown below.



Based on the box plot, determine whether each of the following statements is true, false, or cannot be determined from the information given in the box plot.

|  | True                  | False                 | Cannot Be Determined  |
|--|-----------------------|-----------------------|-----------------------|
| The mean number of tickets sold for Movie X is greater than the mean number of tickets sold for Movie Y.   | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| The median number of tickets sold for Movie X is greater than the median number of tickets sold for Movie Y.   | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| The interquartile range of the number of tickets sold for Movie X is greater than the interquartile range of the number of tickets sold for Movie Y. | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |

Key:

Row 1: Cannot be determined (C)

Row 2: True (T)

Row 3: True (T)

**Scoring Rubric:**

Responses to this item will receive 0-2 points, based upon the following:

**2 points:** CTT

The student has a thorough understanding of how to appropriately use the mean, median, and interquartile range to compare data in box plots. The student knows that the mean cannot be determined from the box plots and correctly compares the median and interquartile range for both data sets.

**1 point:** TTT, FTT

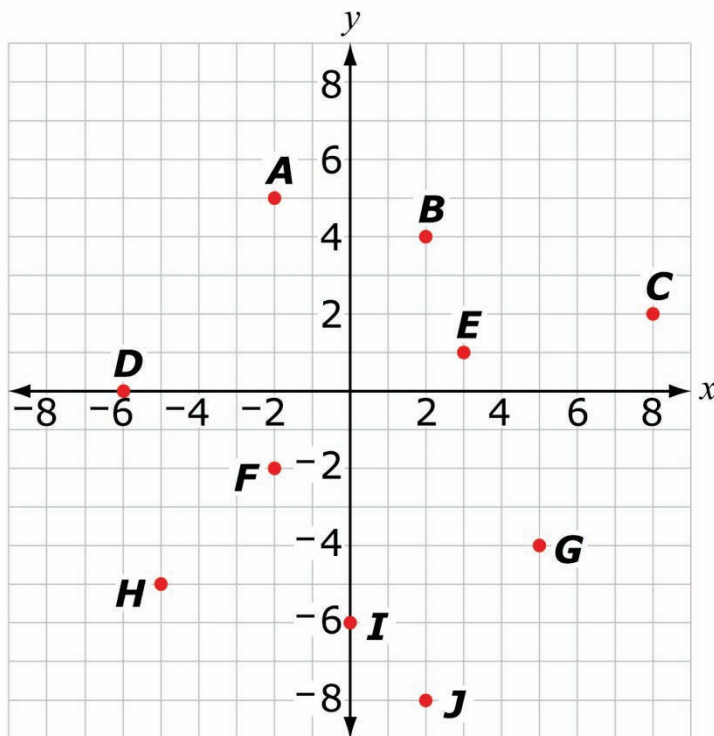
The student has only a basic understanding of how to appropriately use the mean, median, and interquartile range to compare data in box plots. The student correctly compares the median and interquartile range for both data sets but does not realize that the mean cannot be used to compare the data sets.

**0 points:** All other possibilities. The student demonstrates inconsistent understanding of how to appropriately use the mean, median, and interquartile range to compare data in box plots. The student correctly compares either the median or the interquartile range of the two data sets. **OR** The student correctly compares neither the median nor the interquartile range of the two data sets.

MAT.HS.SR.1.OAREI.J.012

|  |   |
|--|---|
| Sample Item ID:  | MAT.HS.SR.1.OAREI.J.012   |
| Grade:   | HS  |
| Claim(s):  | <b>Claim 1: Concepts and Procedures</b><br>Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency. |
| Assessment Target(s):                                    | 1 J: Represent and solve equations and inequalities graphically.  |
| Content Domain:  | Algebra   |
| Standard(s):   | A-REI.12  |
| Mathematical Practice(s):                                | 1, 2, 6   |
| DOK:   | 2   |
| Item Type:   | SR  |
| Score Points:  | 2   |
| Difficulty:  | M   |
| Key:   | G and J only  |
| Stimulus/Source:   |   |
| Target-specific attributes (e.g., accessibility issues): |   |
| Notes:   | Multi-Answer Item   |

The coordinate grid below shows points A through J.





Given the system of inequalities shown below, select all the points that are solutions to this system of inequalities.

$$\begin{cases} x + y < 3 \\ 2x - y > 6 \end{cases}$$

- A       B       C       D       E  
 F       G       H       I       J

*Scoring Rubric for Multi-Part Items:*

*Responses to this item will receive 0-2 points, based on the following:*

**2 points:** The student has a solid understanding of how to determine whether a set of given points is part of the solution to a system of linear inequalities. The student identifies the two correct points, *G* and *J*. The student also recognizes that points that lie in the excluded boundary or on only one of two inequalities are not solutions.

**1 point:** The student has only a basic understanding of how to determine whether a set of given points is part of the solution to a system of linear inequalities. The student identifies the two correct points, *G* and *J*, but does not recognize that points that lie in the excluded boundary or on only one of two inequalities are not solutions and may select points *A* and/or *I* as well.

**0 points:** The student demonstrates inconsistent understanding of how to determine whether a set of given points is part of the solution to a system of linear inequalities. The student identifies no correct points or only one correct point. The student also does not recognize that points that lie in the excluded boundary or on only one of two inequalities are not solutions.

*Rationale for choosing incorrect points:*

*A & I* – The student confuses a point lying on one of the inequalities with being a solution to the system of inequalities.

*B & E* – The student does not understand how to shade sides of the inequalities.

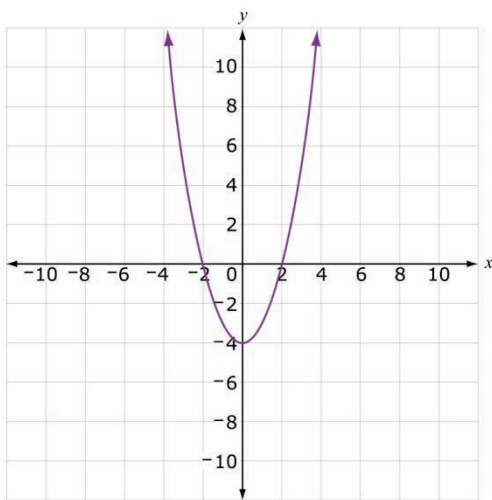
*C* – The student incorrectly shades the side of the first inequality in the system.

*D, F, & H* – The student incorrectly shades the side of the second inequality in the system.

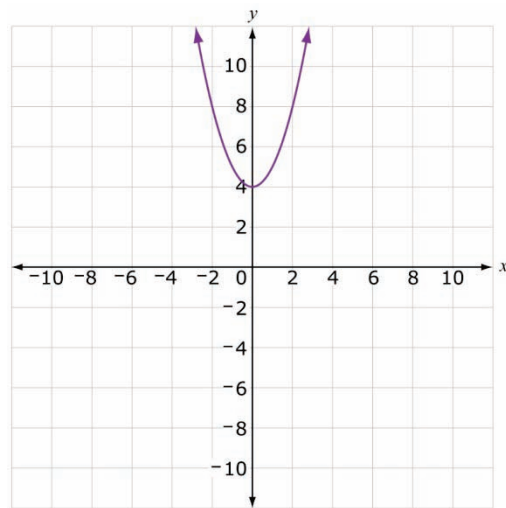
MAT.HS.SR.1.OAREI.J.678

|  |   |
|--|---|
| Sample Item ID:  | MAT.HS.SR.1.OAREI.J.678   |
| Grade:   | HS  |
| Claim(s):  | <b>Claim 1: Concepts and Procedures</b><br>Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency. |
| Assessment Target(s):                                    | 1 J: Represent and solve equations and inequalities graphically.  |
| Content Domain:  | Algebra   |
| Standard(s):   | A-REI.10  |
| Mathematical Practice(s):                                | 1, 2, 4   |
| DOK:   | 2   |
| Item Type:   | SR  |
| Score Points:  | 1   |
| Difficulty:  | M   |
| Key:   | B   |
| Stimulus/Source:   |   |
| Target-specific attributes (e.g., accessibility issues): |   |
| Notes:   |   |

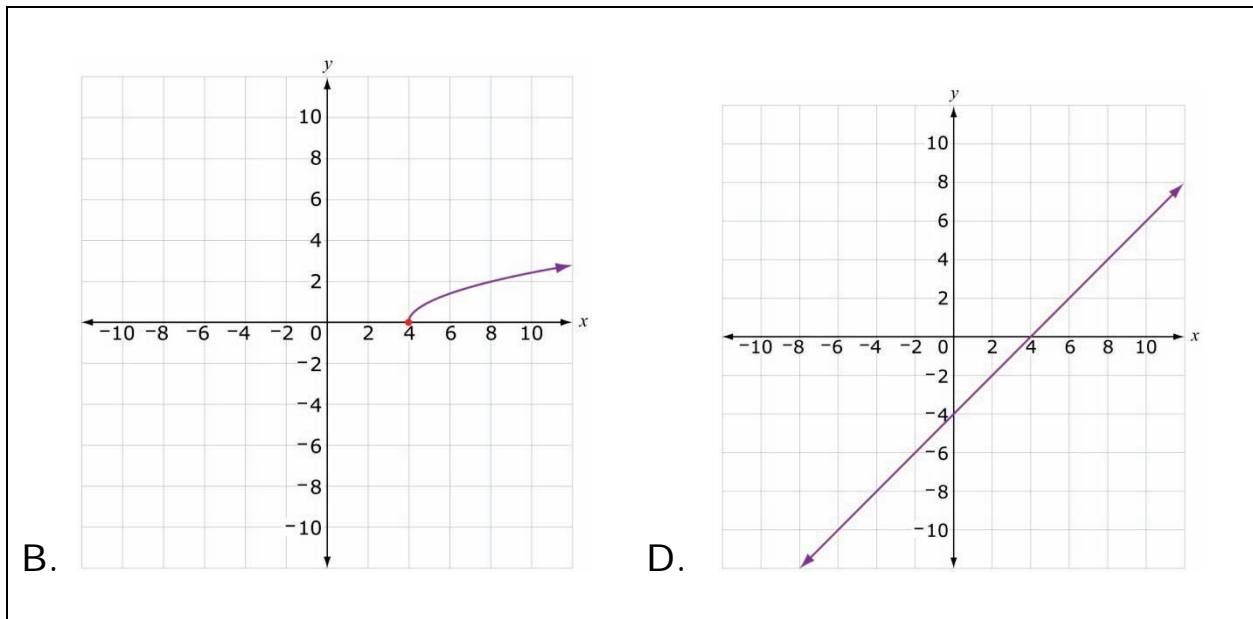
Which graph could represent the solution set of  $y = \sqrt{x - 4}$  ?



A.



C.



*Key and Distractor Analysis:*

- A. Confuses  $y = x^2 - 4$  with  $y = \sqrt{x - 4}$ .
- B. Key
- C. Relates the point (0, 4) on this graph to the 4 under the radicand of the given function.
- D. Confuses radical function with linear function.

## MAT.HS.SR.1.OASSE.E.015

|  |   |
|--|---|
| Sample Item ID:  | MAT.HS.SR.1.OASSE.E.015   |
| Grade:   | HS  |
| Claim(s):  | <b>Claim 1: Concepts and Procedures</b><br>Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency. |
| Assessment Target(s):                                    | 1 E: Write expressions in equivalent forms to solve problems.   |
| Content Domain:  | Algebra   |
| Standard(s):   | A-SSE.3   |
| Mathematical Practice(s):                                | 1, 2, 7   |
| DOK:   | 2   |
| Item Type:   | SR  |
| Score Points:  | 1   |
| Difficulty:  | M   |
| Key:   | 1b and 1d   |
| Stimulus/Source:   |   |
| Target-specific attributes (e.g., accessibility issues): |   |
| Notes:   | Multi-answer item   |

For numbers 1a – 1e, select the two equations with equivalent zeros.

- 1a  $y = x^2 + 14$
- 1b  $y = x^2 + 9x + 14$
- 1c  $y = \left(x - \frac{9}{2}\right)^2 - \frac{25}{4}$
- 1d  $y = (x + 7)(x + 2)$
- 1e  $y = \left(\frac{1}{2}x + 7\right)(2x + 2)$

*Key and Distractor Analysis:*

- 1a. The non-real zeros are  $\pm i\sqrt{14}$ .
- 1b. The zeros are -7 and -2 since the polynomial factors to be the same as in 1d.
- 1c. The zeros are 7 and 2.
- 1d. The zeros are -7 and -2.
- 1e. The zeros are -14 and -1.

HS Mathematics Sample SR Item C1 TE

Both B and D have the same zeros, -7 and -2.

## MAT.HS.TE.1.000NQ.C.083

|  |   |
|--|---|
| Sample Item ID:  | MAT.HS.TE.1.000NQ.C.083   |
| Grade:   | HS  |
| Claim(s):  | <b>Claim 1: Concepts and Procedures</b><br>Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency. |
| Assessment Target(s):                                    | 1 C: Reason quantitatively and use units to solve problems.   |
| Content Domain:  | Number and Quantity   |
| Standard(s):   | N-Q.1   |
| Mathematical Practice(s):                                | 1, 2, 7   |
| DOK:   | 1   |
| Item Type:   | TE  |
| Score Points:  | 1   |
| Difficulty:  | M   |
| Key:   | 20 L, $\frac{1,000 \text{ mL}}{1 \text{ L}}$ , $\frac{0.82 \text{ g}}{1 \text{ mL}}$ , and $\frac{1 \text{ Kg}}{1,000 \text{ g}}$ .                               |
| Stimulus/Source:   | <a href="http://www.simetric.co.uk/si_liquids.htm">http://www.simetric.co.uk/si_liquids.htm</a>   |
| Target-specific Attributes (e.g., accessibility issues): |   |
| Notes:   | TE Template: Select and order   |

The density of kerosene is approximately  $0.82 \frac{\text{g}}{\text{mL}}$ .

Drag a rate or quantity from the box to each blank to calculate the mass, in kilograms, of 20 liters of kerosene.

\_\_\_\_\_ 1 \_\_\_\_\_ × \_\_\_\_\_ 2 \_\_\_\_\_ × \_\_\_\_\_ 3 \_\_\_\_\_ × \_\_\_\_\_ 4 \_\_\_\_\_

|  |  |  |  |
|--|--|--|--|
| 20 L                                   | 820 kg                                 | 820 mL                                 | 2,000 mL                               |
| $\frac{0.82 \text{ g}}{1 \text{ mL}}$  | $\frac{2000 \text{ mL}}{20 \text{ L}}$ | $\frac{1 \text{ L}}{1,000 \text{ mL}}$ | $\frac{1,000 \text{ g}}{1 \text{ kg}}$ |
| $\frac{1 \text{ kg}}{1,000 \text{ g}}$ | $\frac{1 \text{ kg}}{1,000 \text{ L}}$ | $\frac{1,000 \text{ mL}}{1 \text{ L}}$ | $\frac{1,000 \text{ L}}{1 \text{ kg}}$ |

Key:

A correct response to this item will receive 1 point for the following:

The student must choose the following four rates or quantities (order does not matter):

20 L,  $\frac{1,000 \text{ mL}}{1 \text{ L}}$ ,  $\frac{0.82 \text{ g}}{1 \text{ mL}}$ , and  $\frac{1 \text{ Kg}}{1,000 \text{ g}}$ .

One such ordering would be:  $20 \text{ L} \times \frac{1,000 \text{ mL}}{1 \text{ L}} \times \frac{0.82 \text{ g}}{1 \text{ mL}} \times \frac{1 \text{ Kg}}{1,000 \text{ g}}$ .

TE Information:

**Item Code:** MAT.HS.TE.1.000NQ.C.083

**Template:** Select and Order

**Interaction Space Parameters:**

A. The image containing the regions: the four blank lines with numbers:

- [1] "1",
- [2] "2",
- [3] "3",
- [4] "4"

B. *The images for the digital content objects:* 12 ratios with units starting with "20 L" and ending with " $\frac{1,000 \text{ L}}{1 \text{ kg}}$ "; for the scoring data, the objects are labeled A-L starting with the top left (A="20 L") and going across and then down (L=" $\frac{1,000 \text{ L}}{1 \text{ kg}}$ ").

**Scoring Data: (order does not matter)**

{AEIK}=1



## MAT.HS.TE.1.00FBF.N.227

|  |   |
|--|---|
| Sample Item ID:  | MAT.HS.TE.1.00FBF.N.227   |
| Grade:   | HS  |
| Claim(s):  | <b>Claim 1: Concepts and Procedures</b><br>Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency. |
| Assessment Target(s):                                    | 1 N: Build a function that models a relationship between two quantities.  |
| Content Domain:  | Functions   |
| Standard(s):   | F-BF.1  |
| Mathematical Practice(s):                                | 1, 2, 7   |
| DOK:   | 1   |
| Item Type:   | TE  |
| Score Points:  | 2   |
| Difficulty:  | M   |
| Key:   | IMBE  |
| Stimulus/Source:   |   |
| Target-specific attributes (e.g., accessibility issues): |   |
| Notes:   | TE Template: Select and Order   |

A new social networking website was made available. The website had 10 members its first week. Beginning the second week, the creators of the website have a goal to triple the number of members every week.

For *Part A* and *Part B* below, select the appropriate expression for each blank region. To place an expression in a region, click on the expression, move the pointer over the region, and click again to place the expression in the region. Only one expression can be placed in each region. To return all expressions to their original positions, click the Reset button.

|            |             |           |               |               |
|------------|-------------|-----------|---------------|---------------|
| 0          | 1           | 3         | 7             | 10            |
| $3n+7$     | $3n+10$     | $30(n-1)$ | $10(3^{n-1})$ | $3(10^{n-1})$ |
| $f(n-1)+2$ | $f(n-1)+30$ | $3f(n-1)$ | $3f(n-1)+10$  | $f(3n-1)$     |

**Part A**

Determine an explicit formula for  $f(n)$ , the number of members the creators have a goal of getting  $n$  weeks after the website is made available.

$$f(n) = \boxed{\phantom{000000}}$$

**Part B**

Determine a recursive formula for  $f(n)$ .

$$f(n) = \boxed{\phantom{000000}} \quad \text{for } n > \boxed{\phantom{000000}}$$

$$f(1) = \boxed{\phantom{000000}}$$

*Key and Distractor Analysis:*

- A: Assumes general term of recursive formula holds for  $n=1$ .
- B: KEY for Part B,  $n >$ .
- C: Notices number of members triples.
- D: Wildcard.
- E: KEY for Part B,  $f(1) =$ .
- F: Notices correct week 1 value.
- G: Assumes relationship is linear, and assumes formula must involve 3 and 10.
- H: Notices correct week 2 value.
- I: KEY for Part A,  $f(n) =$ .
- J: Notices correct week 2 value, or switches 3 and 10 in key.
- K: Notices correct week 2 value.
- L: Assumes formula must involve product of 3 and 10.
- M: KEY for Part B,  $f(n) =$ .
- N: Assumes 10 must be included in general term of recursive formula.
- O: Places 3 in wrong position.

*Scoring Rubric:*

*Responses to this item will receive 0-2 points, based on the following:*

**2 points:** The student has a solid understanding of how to explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency for writing recursive and explicit functions to describe the relationship between two quantities. The student correctly selects  $f(n)$  for the explicit formula in part A. The student also completely defines the correct recursive formula in part B, selecting the correct  $f(n)$  definition, condition for  $n$ , and initial value for  $f(1)$ .

**1 point:** The student understands how to explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency for writing recursive and explicit functions to describe the relationship between two quantities. The student can identify both the explicit formula in part A and the correct  $f(n)$  definition in part B, but does not correctly identify the condition for  $n$  and/or the initial value for  $f(1)$  in part B.

**0 points:** The student has an inconsistent understanding of how to explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency for writing recursive and explicit functions to describe the relationship between two quantities. The student does not correctly select the  $f(n)$  definitions for both the explicit formula in part A and the  $f(n)$  definition in part B.

*TE Information:*

**Item Code:** MAT.HS.TE.1.00FBF.N.227

**Template:** Select and Order

**Interaction Space Parameters:**

A. *The image containing the regions:* the four blank rectangular areas next to:

[1] " $f(n)=$ ",

[2] " $f(n)=$ ",

[3] "for  $n>$ ",

[4] " $f(1)=$ "

B. *The images for the digital content objects:* 15 numbers and expressions starting with "0" and ending with " $f(3n - 1)$ "; for the scoring data, the objects are labeled A-0 starting with the top left (A="0") and going across and then down (O=" $f(3n - 1)$ ")

**Scoring Data: (X represents incorrect response)**

{ IMBE } = 2

{ IMBX } = 1

{ IMXE } = 1

{ IMXX } = 1

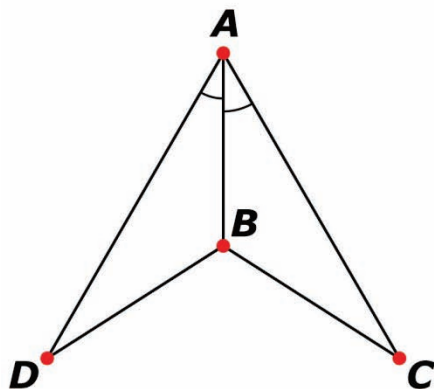
## MAT.HS.TE.1.00GCO.O.470

|  |   |
|--|---|
| Sample Item ID:  | MAT.HS.TE.1.00GCO.O.470   |
| Grade:   | HS  |
| Claim(s):  | <b>Claim 1: Concepts and Procedures</b><br>Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency. |
| Assessment Target(s):                                    | 1 O: Prove geometric theorems.  |
| Content Domain:  | Geometry  |
| Standard(s):   | G-CO.10   |
| Mathematical Practice(s):                                | 2, 3, 7   |
| DOK:   | 2   |
| Item Type:   | TE  |
| Score Points:  | 2   |
| Difficulty:  | L   |
| Key:   | 1. $\angle ABD \cong \angle ABC$<br>2. $\overline{AC} \cong \overline{CE}$  |
| Stimulus/Source:   |   |
| Target-specific attributes (e.g., accessibility issues): |   |
| Notes:   | TEI Template: Select and Order  |

For items 1 and 2, what additional information is required in order to prove the two triangles are congruent using the provided justification?

Use the set of choices in the box below. Select a side or angle and place it in the appropriate region. Only one side or angle can be placed in each region.

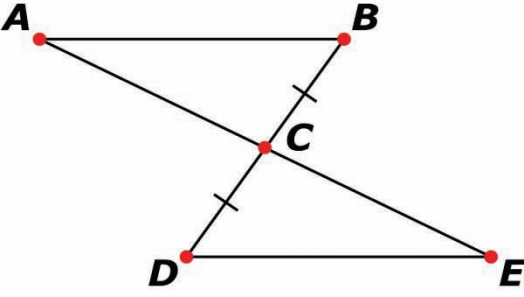
|                 |                 |                 |                 |
|-----------------|-----------------|-----------------|-----------------|
| $\overline{AB}$ | $\overline{AC}$ | $\overline{AD}$ | $\overline{BC}$ |
| $\overline{BD}$ | $\overline{CD}$ | $\overline{CE}$ | $\overline{DE}$ |
| $\angle ABC$    | $\angle ABD$    | $\angle ACB$    | $\angle ADB$    |
| $\angle BAC$    | $\angle CDE$    | $\angle CED$    | $\angle DCE$    |



ASA Postulate

1.

$$\boxed{\phantom{AB}} \cong \boxed{\phantom{AC}}$$



SAS Theorem

2.   $\cong$

*Key for Multi-part Items:*

Each item is scored independently, and will receive 1 point.

*Key*

1.  $\angle ABD \cong \angle ABC$
2.  $\overline{AC} \cong \overline{CE}$

**TE Information**

**Item code:** MAT.HS.TE.1.00GCO.O.270

**TEI Template:** Select and Order

**Interaction Space Parameters:**

- A. *The image containing the pair of blank regions separated by the congruent symbol  $\cong$ :*
  - [1] ASA Postulate  $\langle \text{blank} \rangle \cong \langle \text{blank} \rangle$
  - [2] SAS Theorem  $\langle \text{blank} \rangle \cong \langle \text{blank} \rangle$
- B. *The images for the digital content objects:*  
 The following 8 sides and 8 angles in the lower box:
 
$$\overline{AB}, \overline{AC}, \overline{AD}, \overline{BC}, \overline{BD}, \overline{CD}, \overline{CE}, \overline{DE}$$

$$\angle ABC, \angle ABD, \angle ACB, \angle ADB, \angle BAC, \angle CDE, \angle CED, \angle DCE$$

**Scoring Data:**

Key

1.  $\angle ABD, \angle ABC$
2.  $\overline{AC}, \overline{CE}$

Answer both items correctly {1, 2} = 2  
 One correct item {1} or {2} = 1  
 No correct item = 0

## MAT.HS.TE.1.00SID.P.242

|  |   |
|--|---|
| Sample Item ID:  | MAT.HS.TE.1.00SID.P.242   |
| Grade:   | HS  |
| Claim(s):  | <b>Claim 1: Concepts and Procedures</b><br>Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency. |
| Assessment Target(s):                                    | 1 P: Summarize, represent, and interpret data on a single count or measurement variable.  |
| Content Domain:  | Statistics and Probability  |
| Standard(s):   | S-ID.3  |
| Mathematical Practice(s):                                | 2, 4, 5   |
| DOK:   | 1   |
| Item Type:   | TE  |
| Score Points:  | 2   |
| Difficulty:  | L   |
| Key:   | See Sample Top-Score Response.  |
| Stimulus/Source:   |   |
| Target-specific attributes (e.g., accessibility issues): | Blind or low-vision students may have trouble manipulating the graphics.  |
| Notes:   | See TE information at end.  |

The ages of the students in a certain high school are to be graphed on a set of parallel box plots according to the following:

Set I: All seniors in the school (grade 12)

Set II: All students in the school (grades 9 through 12)

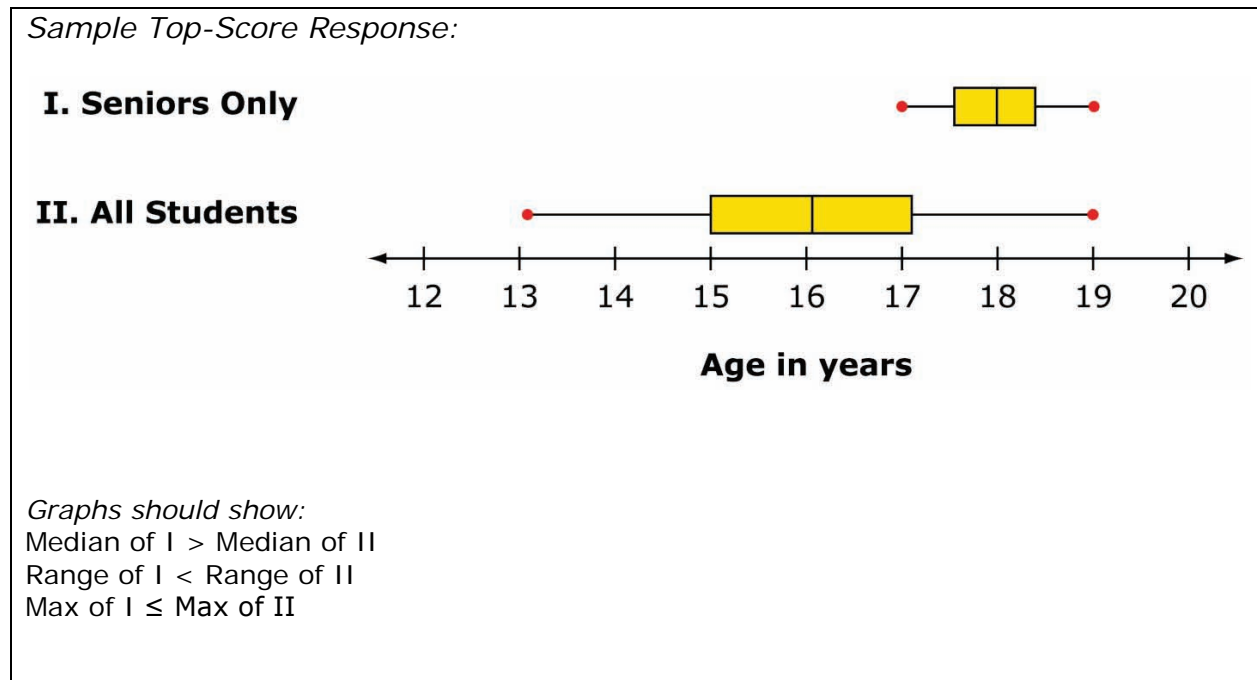
In the figure below, drag each of the two box plots into position above the number line to approximate the ages of the two sets of students. To do this:

- First move each box plot at an appropriate location according to its center.
- Then drag each endpoint to stretch the box plot to represent the spread.

NOTE: There are no outliers in either set.



*Sample Top-Score Response:*



*Scoring Rubric for Multi-Part Items:*

Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student has a solid understanding of how to apply the mathematical concepts of center and spread to compare data sets in context. The student accurately represents the median of Set I as greater than the median of Set II. The student also accurately represents the range of Set I as less than the range of Set II and represents the maximum of Set I as less than or equal to the maximum of Set II.

**1 point:** The student has a basic understanding of how to apply the mathematical concepts of center and spread to compare data sets in context. The student accurately represents the median of Set I as greater than the median of Set II. But the student misrepresents the relationship between the ranges of both sets or between the maximums of both sets.



**0 points:** The student demonstrates an inconsistent understanding of how to apply the mathematical concepts of center and spread to compare data sets in context. The student does not accurately represent the median of Set I as greater than the median of Set II.

*TE Information:*

**Item Code:** MAT.HS.TE.1.00SID.P.242

**Interaction Space Parameters:**

Students will be allowed to click and drag the two box plots horizontally to place them at the appropriate location relative to the number line below. They will then be able to click and drag the dots at the end of each box plot to lengthen or shorten the “whiskers.” The intention is that the min, max, and median of each box plot will “snap” to integer values.

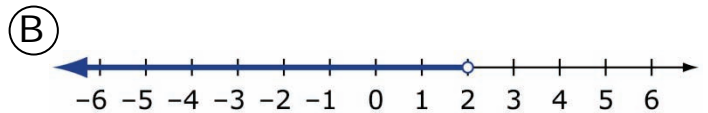
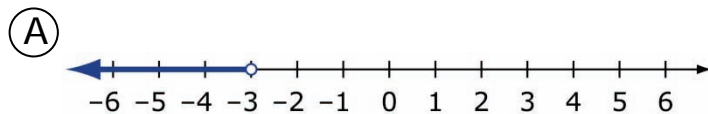
## MAT.HS.TE.1.0AREI.I.088

|   |   |
|---|---|
| Sample Item ID:   | MAT.HS.TE.1.0AREI.I.088   |
| Grade:  | HS  |
| Claim(s):   | <b>Claim 1: Concepts and Procedures</b><br>Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency. |
| Assessment Target(s):                                       | 1 I: Solve equations and inequalities in one variable.  |
| Content Domain:   | Algebra   |
| Standard(s):  | A-REI.3   |
| Mathematical Practice(s):                                   | 4, 6, 7   |
| DOK:  | 1   |
| Item Type:  | TE  |
| Score Points:   | 1   |
| Difficulty:   | M   |
| Key:  | 1-F, 2-B, 3-F   |
| Stimulus/Source:  |   |
| Target-specific attributes<br>(e.g., accessibility issues): |   |
| Notes:  | TEI Template: Connections   |

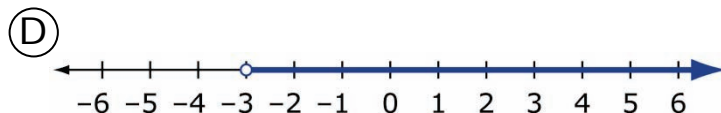
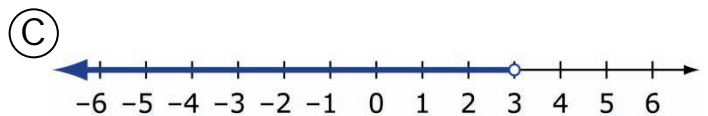
Match each inequality in items 1 – 3 with the number line in items A – F that represent the solution to the inequality.

To connect an inequality to its number line, first click the inequality. Then click the number line it goes with. A line will automatically be drawn between them.

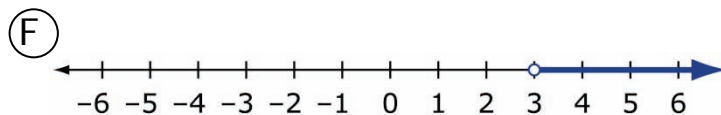
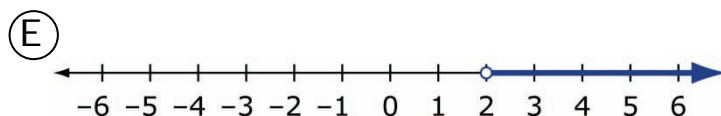
①  $-4x < -12$



②  $2(x + 2) < 8$



③  $5 - 2x < 2 - x$



*Key and Distractor Analysis:*

1. Key F; Students that match this inequality correctly have demonstrated an understanding of how the inequality symbol is affected when dividing by a negative number.
2. Key B; Students that match this inequality correctly have demonstrated an

understanding of how to apply the distributive property when solving multi-step problems.

3. Key F; Students that match this inequality correctly have demonstrated an understanding of how to solve inequalities with variable terms on both sides.

**TE Information**

**Item Code:** MAT.HS.TE.1.0AREI.I.088

**TEI Template:** Connections

**Prompt:** Match the inequalities in items 1 – 3 with their number line solutions in A – F.

**Interaction Space Parameters:**

A. Three equations in first region:

Referred to as:

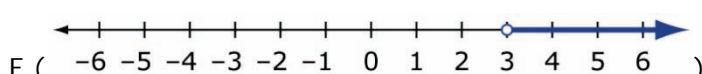
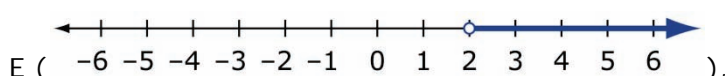
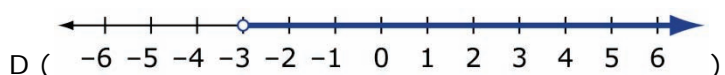
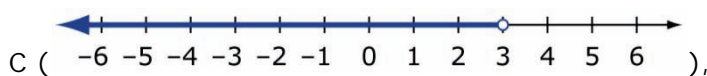
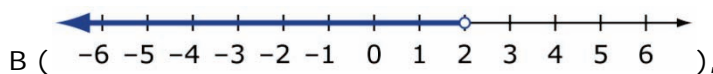
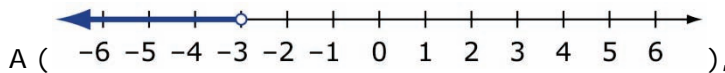
1  $(-4x < -12)$ ,

2  $(2(x + 2) < 8)$ ,

3  $(5 - 2x < 2 - x)$

B. Six images in second region:

Referred to as:



C. True

D. True

**Scoring Data:**

{1-F, 2-B, 3-F} {0 errors=1}

**Scoring Rule Explanation:**

Based on the scoring rule and the scoring data for this particular item, students that properly connect each inequality to their correct number lines will get a score of 1. All other connections will receive a score of 0.

**MAT.HS.TE.1.OAREI.J.087**

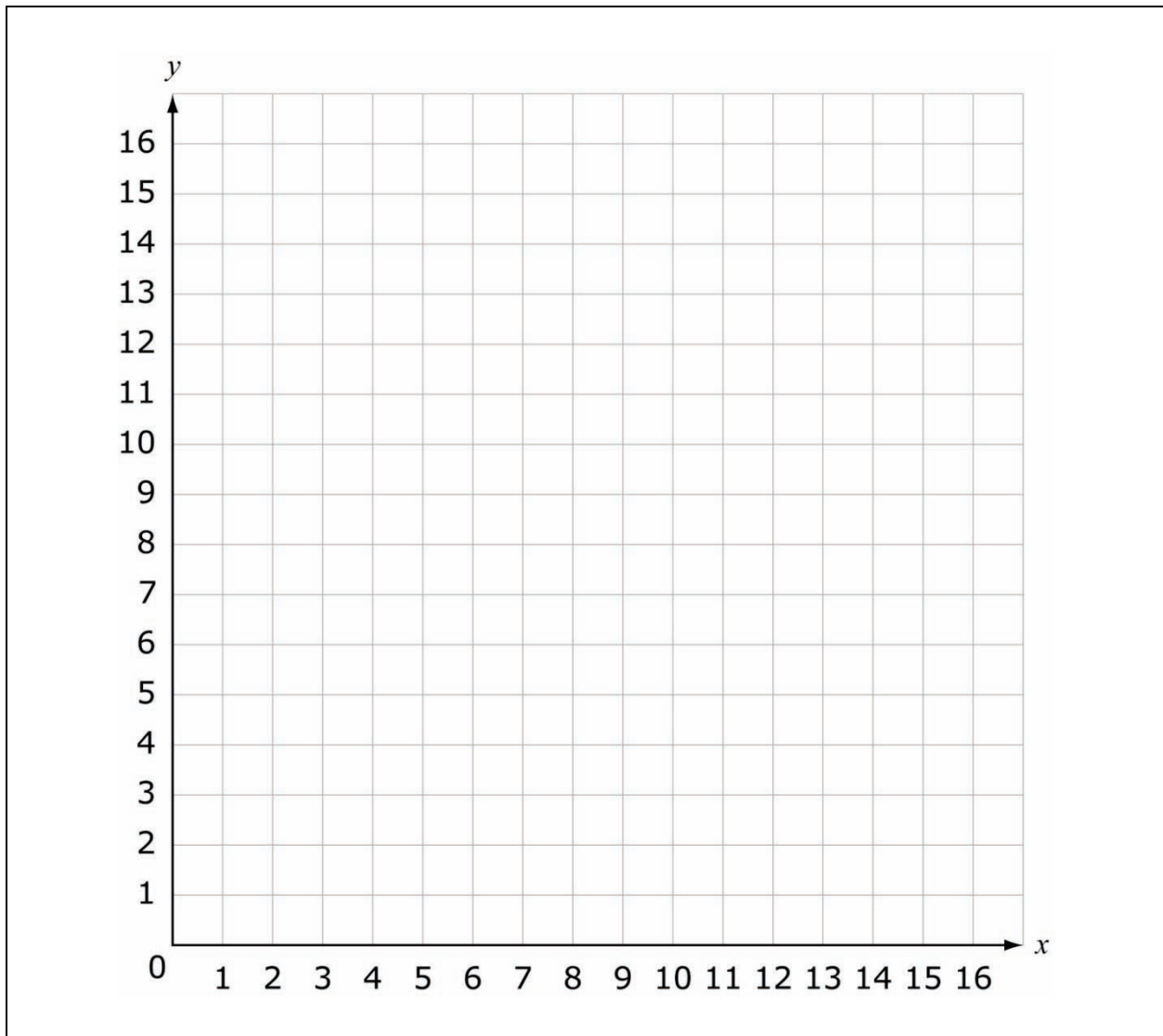
|  |   |
|--|---|
| Sample Item ID:  | MAT.HS.TE.1.OAREI.J.087   |
| Grade:   | HS  |
| Claim(s):  | <b>Claim 1: Concepts and Procedures</b><br>Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency. |
| Assessment Target(s):                                    | 1 J: Represent and solve equations and inequalities graphically.  |
| Content Domain:  | Algebra   |
| Standard(s):   | A-REI.12  |
| Mathematical Practice(s):                                | 1, 5  |
| DOK:   | 2   |
| Item Type:   | TE  |
| Score Points:  | 2   |
| Difficulty:  | M   |
| Key:   | See Sample Top-Score Response.  |
| Stimulus/Source:   |   |
| Target-specific attributes (e.g., accessibility issues): |   |
| Notes:   | TEI Template: Multi-lines; then select  |

Graph this system of inequalities below on the given coordinate grid.

$$\begin{cases} x + y \geq 12 \\ 20x + 30y \leq 300 \end{cases}$$

To create a line, click in the grid to create the first point on the line. To create the second point on the line, move the pointer and click. The line will be automatically drawn between the two points. Use the same process to create additional lines.

When both inequalities are graphed, select the region in your graph that represents the solution to this system of inequalities. To select a region, click anywhere in the region. To clear a selected region, click anywhere in the selected region.



*Scoring Rubric:*

*Responses to this item will receive 0-2 points, based on the following:*

**2 points:** The student has a solid understanding of how to solve a system of inequalities graphically. The student correctly graphs both inequalities and identifies the correct region that represents the solution to the system, region IV.

**1 point:** The student has some understanding of how to solve a system of inequalities graphically. The student correctly graphs both inequalities but does not identify the correct region that represents the solution to the system. **OR** The student incorrectly graphs one or both inequalities but identifies the correct region that represents the solution to the incorrectly graphed system.

**0 points:** The student demonstrates inconsistent understanding of how to solve a system

of inequalities graphically. The student does not correctly graph both inequalities and/or does not identify the correct region that represents the solution to the system.

*TE Information:*

**Item Code:** MAT.HS.TE.1.0AREI.J.087

**Template:** Multi-lines; then select

**Interaction Space Parameters:**

- A: False (do not use default grid)
- B: Bottom-left corner is (0,0); top-right corner is (15,15); grid line increment size is 1; axes are labeled with X and Y
- C: Label each grid increment
- D: True (support snap-to behavior)
- E: Limit = true
- F: Maximum number of lines is 2
- G: Solid lines
- H: Limit = true
- I: Maximum number of sections that can be selected is 1

**Scoring Data:**

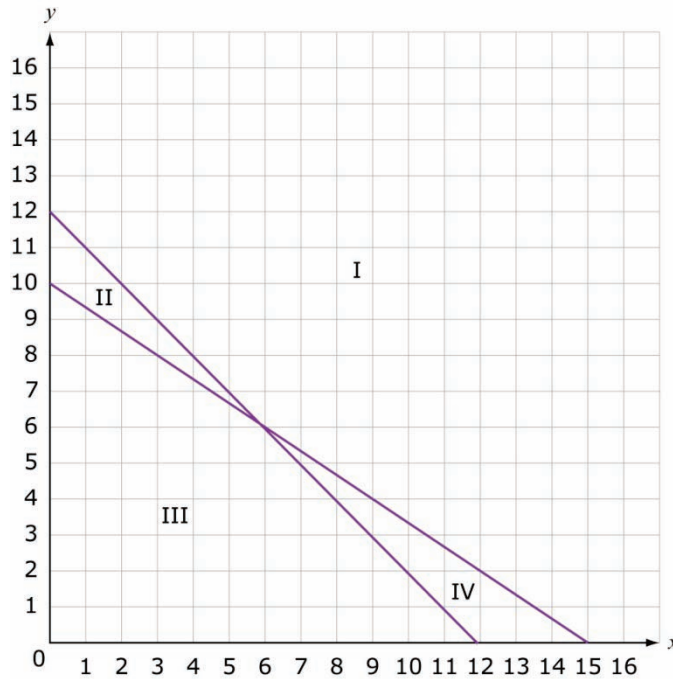
Line 1:  
 x-Intercept  
 Consider = true  
 (12,0)  
 0  
 y-Intercept  
 Consider = true  
 (0,12)  
 0  
 Slope  
 Consider = false  
 Line 2:  
 x-Intercept  
 Consider = true  
 (15,0)  
 0  
 y-Intercept  
 Consider = true  
 (0,10)  
 0  
 Slope  
 Consider = false

**Grid section: II**

*The figure below represents how the four sections would be labeled: section I, section II, section III, and section IV.*

*One line contains the points (12, 0) and (0, 12).*

The other line contains the points  $(15, 0)$  and  $(0, 10)$ .



**Scoring Rule Explanation:**

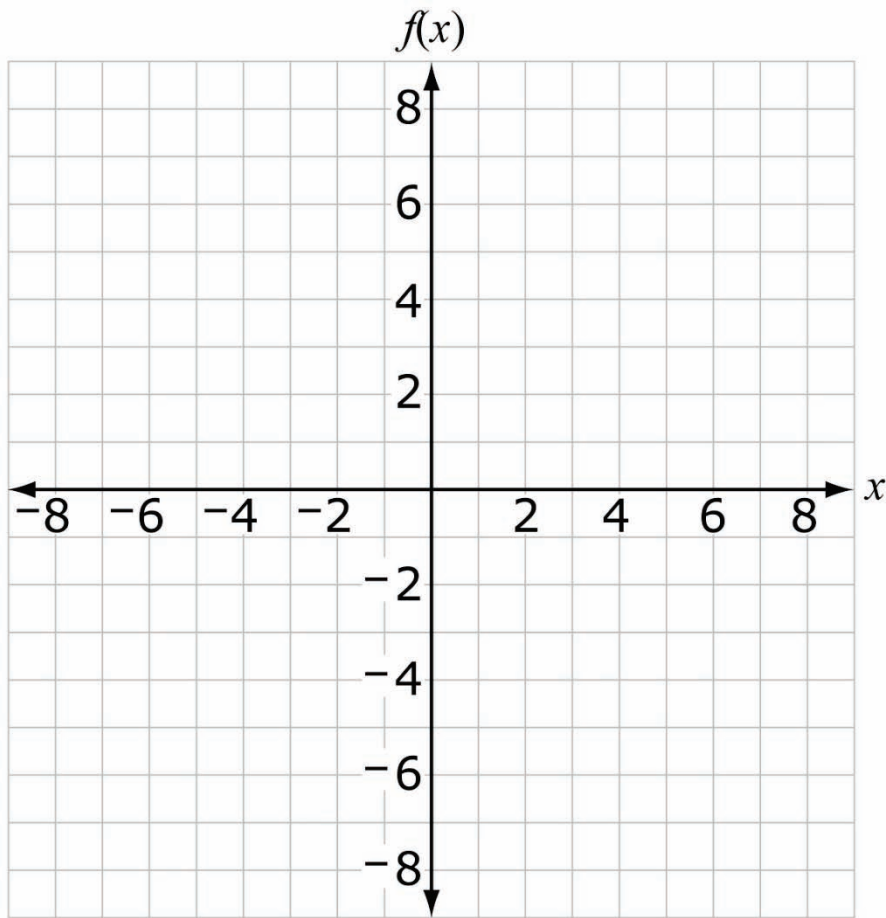
Based on the scoring rule and the scoring data for this particular item, students that create two lines representing  $y = -x + 12$  and  $y = -\frac{2}{3}x + 10$  and select the section of the plane represented by the intersection of  $y \geq -x + 12$  and  $y \leq -\frac{2}{3}x + 10$  (IV, above) will receive 1 point. All other responses will receive 0 points.



**MAT.HS.TE.2.00FBF.B.046**

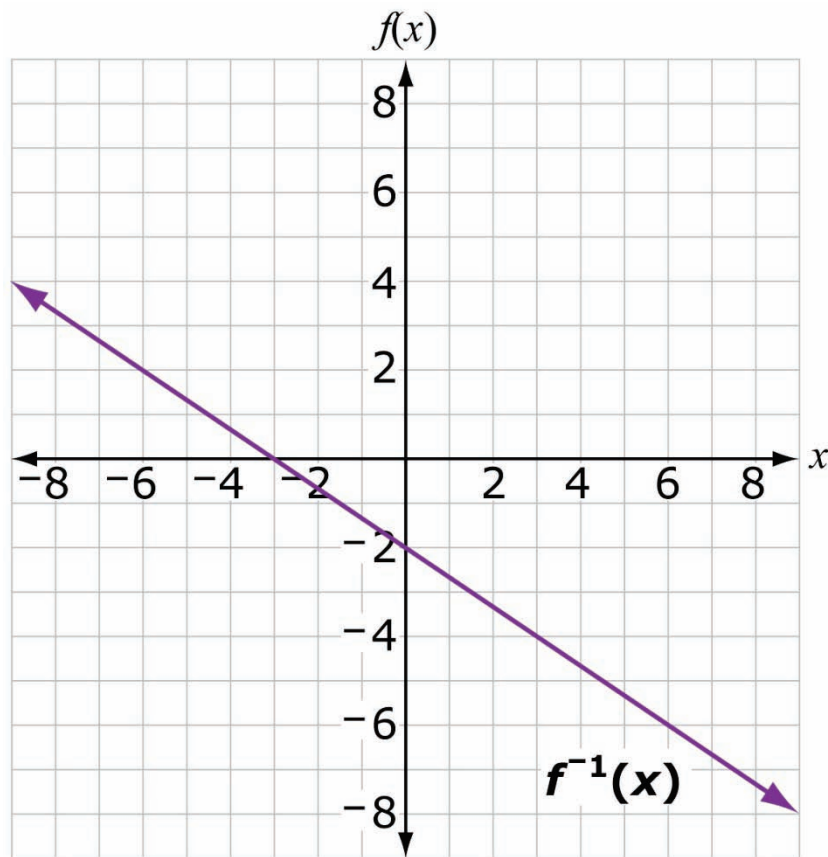
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|--|--|
| Sample Item ID:  | MAT.HS.TE.2.00FBF.B.046  |
| Grade:   | HS   |
| Primary Claim:   | <b>Claim 2: Problem Solving</b><br>Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies. |
| Secondary Claim(s):                                      | Claim 1: Concepts and Procedures<br>Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.                 |
| Primary Content Domain:                                  | Functions  |
| Secondary Content Domain(s):                             |  |
| Assessment Target(s):                                    | 2 B: Select and use appropriate tools strategically.   |
| Standard(s):   | F-BF.4   |
| Mathematical Practice(s):                                | 1, 5   |
| DOK:   | 2  |
| Item Type:   | TE   |
| Score Points:  | 1  |
| Difficulty:  | M  |
| Key:   | See Sample Top-Score Response.   |
| Stimulus/Source:   |  |
| Target-specific attributes (e.g., accessibility issues): |  |
| Notes:   | TE Template: Single Line   |

Draw the graph of the inverse of  $f(x) = -\frac{3}{2}x - 3$  on the coordinate grid below.



[To create a line, click in the coordinate grid below to create the first point on the line. To create the second point on the line, move the pointer and click.]

Sample Top-Score Response:



Correct line graphs will receive 1 point.

Key: line containing y-intercept  $(0, -2)$  and slope of  $-\frac{2}{3}$

*TE Information:*

**Item Code:** MAT.HS.TE.3.00FBF.E.046

**Template:** Single Line

**Interaction Space Parameters:**

- A. False
- B. Grid centered at (0, 0); point in bottom-left corner is (-8, -8); point in top-right corner is (8, 8); grid increment size is one unit; coordinate axes are displayed and labeled with  $x$  and  $f(x)$ .
- C. Make grid visible
- D. Label first and last grid increment
- E. False
- F. N/A
- G. True
- H. Draw extended line

**Scoring Data:**

Start Point

A: Do not consider

End Point

A: Do not consider

x-Intercept

A: Do not consider

y-Intercept

A: Consider

B: -2

C: 0

Slope

A: Consider

B:  $-\frac{2}{3}$

C: 0

**Scoring Rule Explanation:**

Based on the scoring rule and the scoring data for this particular item, students that create a line with y-intercept (0, -2) and slope of  $-\frac{2}{3}$  will receive 1 point. All other lines will receive 0 points.